When you are done with your homework you should be able to…

- π Find the slope of the tangent line to a curve at a point
- π Use the limit definition to find the derivative of a function
- π Understand the relationship between differentiability and continuity

Warm-up: Find the following limits.

1. $\lim_{x \to 0} \frac{3x}{x^2 + 2x}$

2. $\lim_{x \to 0} \frac{\frac{1}{x+4} - \frac{1}{4}}{x}$

3. $\lim_{\Delta x \to 0} \frac{(x + \Delta x)^2 - x^2}{\Delta x}$

DEFINITION OF TANGENT LINE WITH SLOPE *m*

If f is defined on an open interval containing *c*, and if the limit

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(c+\Delta x) - f(c)}{\Delta x} = m$$

exists, then the line passing through $(c, f(c))$ with slope m is the **tangent line** to the graph of f at the point $(c, f(c))$.

The slope of the tangent line to the graph of f at the point $(c, f(c))$ is also called the **slope of the graph of *f* at *x* = *c*.

Example 1: Find the slope of the graph of $f(x) = 6 - x^2$ at the point $(1,5)$.

DEFINITION FOR VERTICAL TANGENT LINES

If f is continuous at *c* and

$$\lim_{\Delta x \to 0} \frac{f(c+\Delta x) - f(c)}{\Delta x} = \infty \quad \text{or} \quad \lim_{\Delta x \to 0} \frac{f(c+\Delta x) - f(c)}{\Delta x} = -\infty$$

the vertical line $x = c$ passing through the point $(c, f(c))$ is a **vertical tangent line** to the graph of f.

DEFINITION OF THE DERIVATIVE OF A FUNCTION

The **derivative** of f at x is given by

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

provided the limit exists. For all x for which this limit exists, f' is a function of x.

The process of finding the derivative of a function is called

___________________.

A function is _____________________ at x if its derivative exists at x and

is ____________________ on an open interval (a, b) if it is differentiable at every point in the interval.

NOTATION FOR THE DERIVATIVE OF $y = f(x)$:

Example 2: Find the derivative of $f(x) = 4 - x^3$ using the limit process.

ALTERNATIVE LIMIT FORM OF THE DERIVATIVE

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

This form of the derivative requires that the one-sided limits

$\lim_{x \to c^-} \frac{f(x) - f(c)}{x - c}$ and $\lim_{x \to c^+} \frac{f(x) - f(c)}{x - c}$ exist and are equal.

Example 3: Is the function $f(x) = x^{2/3}$ differentiable at $x = 0$?

THEOREM: DIFFERENTIABILITY IMPLIES CONTINUITY

If f is differentiable at $x = c$, then f is continuous at $x = c$.