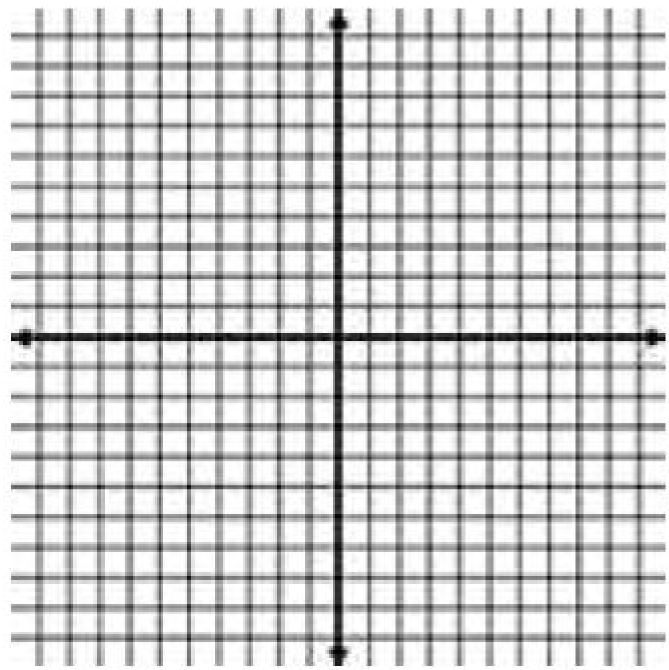
When you finish your homework you should be able to…

- Estimate a limit using a numerical or graphical approach.
- Learn different ways that a limit can fail to exist.
- Study and use a formal definition of limit.

Consider the function $f(x)=\frac{x^3-8}{x-2}$

Let's graph the function:



Now let's examine the limit of the function as *x* approaches 2 using a table. This is written as $\lim_{x\to 2} f(x)$, or specifically $\lim_{x\to 2}\frac{x^3-8}{x-2}$ for our function.

x	1.9	1.99	1.999	2.001	2.01	2.1
$f(x)$						

COMMON TYPES OF BEHAVIOR ASSOCIATED WITH NONEXISTENCE OF A LIMIT

1. $f(x)$ approaches a different number from the right side of *c* than it approaches from the left side.
2. $f(x)$ increases or decreases without bound as *x* approaches *c*.
3. $f(x)$ oscillates between two fixed values as *x* approaches *c*.

Complete the table and use the result to estimate the limit.

1. $\lim_{x \to -3} \frac{\sqrt{1-x}-2}{x+3}$

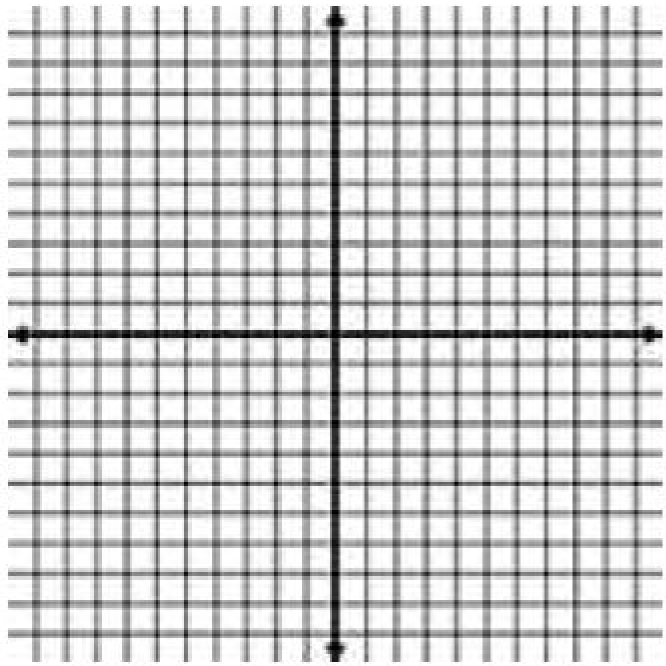
x	-3.1	-3.01	-3.001	-2.999	-2.99	-2.9
$f(x)$						

2. $\lim_{x \to 0} \cos \frac{1}{x}$

x	$1/\pi$	$1/2\pi$	$1/3\pi$	$1/4\pi$	$1/5\pi$	$1/6\pi$
$f(x)$						

3. Consider the function $f(x)=\dfrac{x}{x-3}$

Let's graph the function:

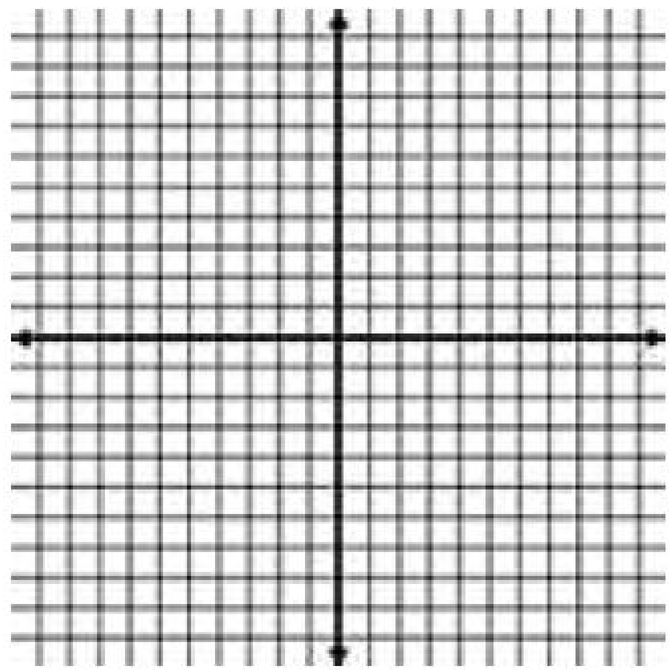


x	2.9	2.99	2.999	3.001	3.01	3.1
$f(x)$						

What observations would you make about the behavior of this function at the asymptote?

4. Consider the function $g(x)=\begin{cases}\sin x, & x\le 0\\ 1-\cos x, & 0\le x\le \pi\\ \cos x, & x>\pi\end{cases}$

Let's graph the function:



Now let's identify the values of c for which the $\lim_{x\to c} g(x)$ exists.

DEFINITION OF LIMIT

Let *f* be a function defined on an open interval containing *c* (except possibly at *c*) and let *L* be a **real number**. The statement

$$\lim_{x \to c} f(x) = L$$

means that for each small positive number epsilon, denoted ε, there exists a small positive number delta, denoted δ, such that if

$$0 < |x - c| < \delta, \text{ then } |f(x) - L| < \varepsilon.$$

5. Find the limit *L*. Then find $\delta > 0$ such that $|f(x) - L| < 0.01$ whenever $|x - c| < \delta$.

$$\lim_{x \to 5} \left(x^2 + 4\right)$$