When you are done with your homework you should be able to…

- Use the Log Rule for Integration to integrate a rational function
- Integrate trigonometric functions

Warm-up:

1. Differentiate the following functions with respect to x.

a. $y = x\ln 5x$

$\frac{dy}{dx} = 1\ln 5x + x\cdot\frac{5}{5x}$

$\boxed{\frac{dy}{dx} = 1 + \ln 5x}$

b. $\ln(xy) = \ln(x+y)$.

$\frac{d}{dx}(\ln x + \ln y) = \frac{d}{dx}\ln(x+y)$

$\frac{1}{x} + \frac{1}{y}\cdot\frac{dy}{dx} = \frac{1}{x+y}\cdot\left(1+\frac{dy}{dx}\right)$

$xy(x+y)\left(\frac{1}{x} + \frac{1}{y}\frac{dy}{dx}\right) = \left(\frac{1}{x+y} + \frac{1}{x+y}\cdot\frac{dy}{dx}\right)xy(x+y)$

$y(x+y) + x(x+y)\frac{dy}{dx} = xy + xy\frac{dy}{dx}$

$x(x+y)\frac{dy}{dx} - xy\frac{dy}{dx} = xy - y(x+y)$

$\frac{d}{dx}(x+y)$

$\frac{dx}{dx} + \frac{dy}{dx}$

$1 + \frac{dy}{dx}$

$\frac{dy}{dx}\left(x\left[(x+y)-y\right]\right) = y(x-x-y)$

$\frac{dy}{dx} = -\frac{y^2}{x}$

THEOREM: LOG RULE FOR INTEGRATION

Let u be a differentiable function of x.

1. $\int \frac{1}{x}dx = \ln|x| + C$

2. $\int \frac{1}{u}du = \ln|u| + C$

$\frac{d}{dx}\ln|x| = \frac{1}{x}$

$\frac{1}{u}\,du = \frac{du}{u} = u^{-1}\,du$

Example 1: Find or evaluate the integral.

a. $\int \frac{10}{x}dx$ $= 10\int \frac{1}{x}\,dx$

$= 10\ln|x| + C$

$= \ln|x|^{10} + C$

$= \boxed{\ln x^{10} + C}$

b. $\int \frac{x^2}{\sqrt{5-x^3}}dx$

c. $\int \frac{x}{\sqrt{1-x^2}}dx$

d. $\int_e^{e^2} \frac{dx}{x \ln x}$

e. $\int_1^e \frac{(1+\ln x)^2 \, dx}{x}$

f. $\int \frac{1}{x^{2/3}\left(1+x^{1/3}\right)} dx$

g. $\int \frac{x^3 - 6x - 20}{x+5} dx$

h. $\int \tan\theta d\theta$

i. $\int \cot\theta d\theta$

j. $\int \sec\theta d\theta$

k. $\int \csc\theta d\theta$

INTEGRALS OF THE SIX BASIC TRIGONOMETRIC FUNCTIONS

$\int \sin u\,du = \_\_\_\_\_$ $\int \cos u\,du = \_\_\_\_\_$

$\int \tan u\,du = \_\_\_\_\_$ $\int \cot u\,du = \_\_\_\_\_$

$\int \sec u\,du = \_\_\_\_\_$ $\int \csc u\,du = \_\_\_\_\_$

Example 2: Solve the differential equation.

a. $y' = \dfrac{x+1}{x-1}$

b. $r' = \theta \tan \theta^2$

Example 3: The demand equation for a product is $p = \frac{90,000}{400+3x}$. Find the average price on the interval $40 \le x \le 50$.