MATH 150/GRACEY NAME________________________

WORKSHEET/3.7

Guidelines for Solving Applied Minimum and Maximum Problems

1. Identify all given quantities and all quantities to be determined. MAKE A SKETCH!!!
2. Write a **primary equation** for the quantity that is to be maximized or minimized.
3. Reduce the primary equation to one having a single independent variable. You may need to use secondary equations relating the independent variables of the primary equation.
4. Determine the **feasible** domain of the primary equation.
5. Determine the desired maximum or minimum value using the techniques learned in 3.1-3.4.
6. State your conclusion in words.

1. Find two positive numbers that satisfy the following requirements: The sum of the first number squared and the second is 27 and the product is a maximum.

1) Analysis

Let a be the 1st #

Let b be the 2nd #

2) Primary Equation

$P(a,b) = ab$

3) Reduce Primary to 1 ind. var.

$a^2 + b = 27$

$b = 27 - a^2$

$P(a) = a(27 - a^2)$

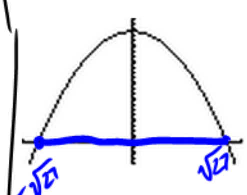
$P(a) = 27a - a^3$

4) Feasible domain

$a > 0, \; b > 0$

$b = 27 - a^2$

$27 - a^2 > 0$



$0 < a < 5.2$

$b > 0$

5) Optimize and verify

i) Find C.N.

$P(a) = 27a - a^3$

$P'(a) = 27 - 3a^2$

$0 = 27 - 3a^2$

$3a^2 = 27$

$a^2 = 9$

$a = \cancel{\pm} 3$

$a = 3$

ii) Use 2nd deriv. test to verify $a = 3$ yields the max product

$P'(a) = 27 - 3a^2$

$P''(a) = -6a$

$P''(3) = -6(3) < 0 \Rightarrow$ rel. max

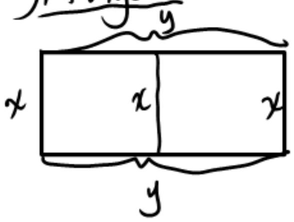
so $a = 3$, $b = 27 - (3)^2 = 18$

6) Conclusion

The numbers are 3 and 18.

2. A woman has two dogs that do not get along. She has a theory that if they are housed in kennels right next to each other, they'll get used to each other. She has 200 feet of fencing with which to enclose two adjacent rectangular kennels. What <u>dimensions</u> should be used so that the enclosed area will be a maximum?

1) Analysis



2) Primary Equation

$P(x,y) = xy$

3) Reduce Primary

$3x + 2y = 200$

$y = \frac{200-3x}{2}$

$y = 100 - \frac{3}{2}x$

$P(x) = x\left(100 - \frac{3}{2}x\right)$

$P(x) = 100x - \frac{3}{2}x^2$

4) Domain

$x > 0,\ y > 0$

$100 - \frac{3}{2}x > 0$

$100 > \frac{3}{2}x$

$\frac{200}{3} > x$

$0 < x < \frac{200}{3},\ y > 0$

5) Optimize

i) Critical #

$P(x) = 100x - \frac{3}{2}x^2$

$P'(x) = 100 - 3x$

$0 = 100 - 3x$

$x = \frac{100}{3}$

ii) Verify $x = \frac{100}{3}$ yields a max

$P'(x) = 100 - 3x$

$P''(x) = -3 < 0$

$\frac{100}{3}$ yields a rel. max

$y = 100 - \frac{3}{2}x$

$y = 100 - \frac{3}{2}\left(\frac{100}{3}\right)$

$y = 100 - 50$

$y = 50$

6) Conclusion

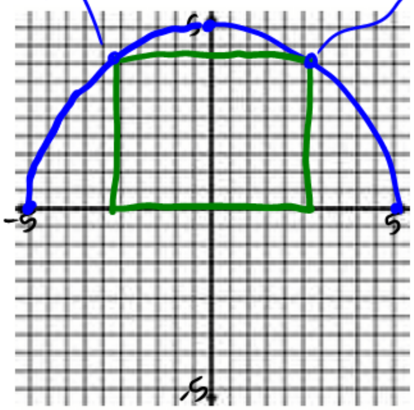
The dimensions which yield the maximum area are $\frac{100}{3}$ ft x 50 ft.

1) Analysis

3. A rectangle is bounded by the x-axis and the semi-circle $y=\sqrt{25-x^2}$. What length and width should the rectangle have so that its area is a maximum?

$(-x, y) = (-x, \sqrt{25-x^2})$

$(x, y) = (x, \sqrt{25-x^2})$



base : $x-(-x)=2x$

height : y

2) Primary Equation

$A(x,y) = 2xy$

3) Reduce Primary

$A(x) = 2x\sqrt{25-x^2}$

4) Feasible Domain

$0 < x < 5$, $0 < y < 5$

5) Optimize and verify

i) C.N.

$A(x) = 2x(25-x^2)^{1/2}$

$A'(x) = 2(25-x^2)^{1/2} + \cancel{2}x\left[\frac{1}{\cancel{2}}(25-x^2)^{-1/2}\cdot(-2x)\right]$

$A'(x) = 2(25-x^2)^{-1/2}\left[(25-x^2)^1 - x^2\right]$

$A'(x) = \dfrac{2(25-2x^2)}{(25-x^2)^{1/2}}$

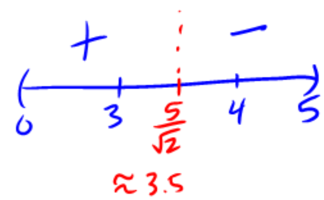
$0 = 25-2x^2$

$2x^2 = 25$

$x^2 = \frac{25}{2}$

$x = \cancel{\pm}\frac{5}{\sqrt{2}}$

ii) verify $x = \frac{5}{\sqrt{2}} \approx 3.5$ yields a max



$A'(3) > 0$, $A'(4) < 0$

$y = \sqrt{25-x^2}$, $x = \frac{5}{\sqrt{2}}$

$y = \sqrt{25-\left(\frac{5}{\sqrt{2}}\right)^2}$

$y = \sqrt{25-\frac{25}{2}}$

$y = \sqrt{\frac{50-25}{2}}$

$y = \sqrt{\frac{25}{2}}$

$y = \frac{5}{\sqrt{2}}$

base : $2x = \frac{10}{\sqrt{2}}$

6) Conclusion

The base is $\frac{10}{\sqrt{2}}$ units and the height is $\frac{5}{\sqrt{2}}$ units.

4. Two factories are located at the coordinates $(-x, 0)$ and $(x, 0)$, with their power supply located at $(0, h)$. Find y such that the total length of power line is a minimum.