When you are done with your homework you should be able to…

- π Understand and use Rolle's Theorem
- π Understand and use the Mean Value Theorem

Warm-up: Locate the global extrema of the function $f(x)=x^3-12x$ on the closed interval $[0,4]$.

$f'(x)=3x^2-12$
$0=3x^2-12$
$x^2=4$
$x=\pm 2$
$c=2$

$f(2)=-16$
$f(0)=0$
$f(4)=16$

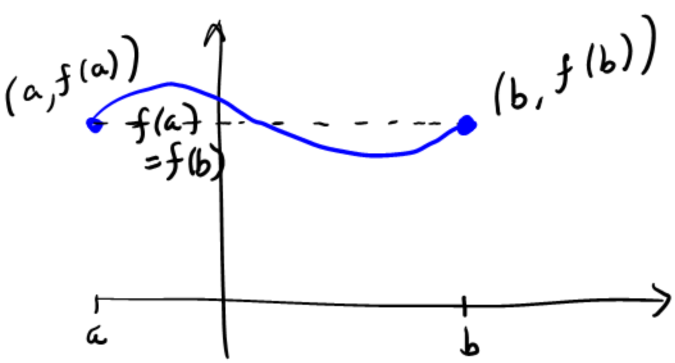
The global min. is -16 at $(2,-16)$.
The global max. is 16 at $(4,16)$

ROLLE'S THEOREM

The __Extreme__ Value Theorem states that a continuous function on a closed interval __$[a,b]$__ must have both a __minimum__ and a __maximum__. Both of these values, however, can occur at the __endpoints__. Rolle's Theorem gives conditions that guarantee the existence of an extreme value in the __interior__ of a closed interval.

THEOREM: ROLLE'S THEOREM

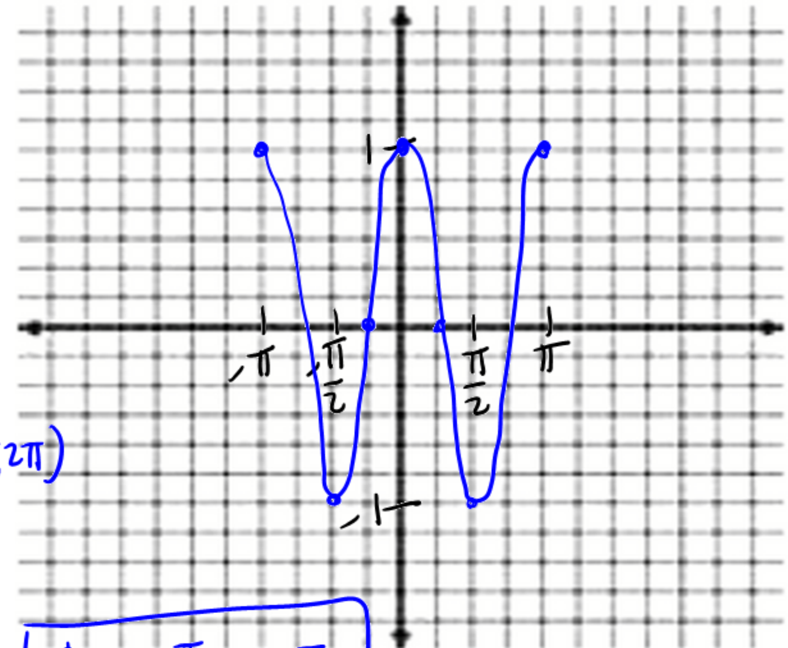
Let f be continuous on the closed interval $[a,b]$ and differentiable on the open interval (a,b). If $f(a)=f(b)$ then there is at least one number c in (a,b) such that $f'(c)=0$.



f is cont. on $[a,b]$ ✓
f is differentiable on (a,b) ✓
$f(a)=f(b)$ ✓
So Rolles Thm may be applied.

Example 1: Determine whether Rolle's Theorem can be applied to f on the closed interval $[a,b]$. If Rolle's Theorem can be applied, find all values of c in the open interval (a,b) such that $f'(c)=0$.

a. $f(x)=\cos 2x,\ [-\pi,\pi]$



Conditions:

f is cont. on $[-\pi,\pi]$ ✓

f is differentiable on $(-\pi,\pi)$ ✓

$f(-\pi) = 1 = f(\pi)$ ✓

Rolles Thm can be applied Yaay!

Apply Rolle's Thm

$f'(x) = -2\sin 2x$

$0 = -2\sin 2x$

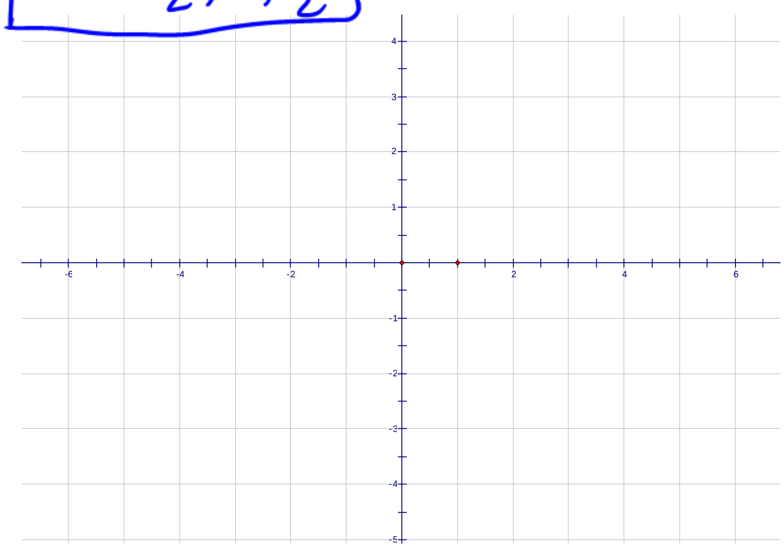
$0 = \sin 2x$ $(-2\pi, 2\pi)$

$2x = -\pi, 0, \pi$

$x = -\frac{\pi}{2}, 0, \frac{\pi}{2}$

$\boxed{c = -\frac{\pi}{2}, 0, \frac{\pi}{2}}$

b. $f(x)=x^2-5x+4,\ [1,4]$



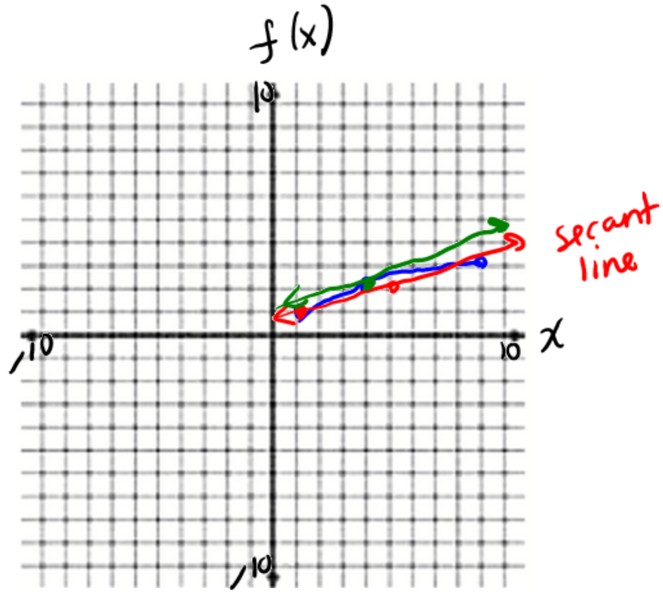
THEOREM: THE MEAN VALUE THEOREM

If f is continuous on the closed interval $[a,b]$, and differentiable on the open interval (a,b), then there exists a number c in (a,b) such that

$$f'(c) = \frac{f(b) - f(a)}{b-a}$$

Example 2: Consider the function $f(x) = \sqrt{x}$ on the closed interval from $[1,9]$.

a. Graph the function on the given interval.



b. Find and graph the secant line through the endpoints on the same coordinate plane.

$f(x) = \sqrt{x}$

$a = 1, \; f(a) = f(1) = \sqrt{1} = 1$

$b = 9, \; f(b) = f(9) = \sqrt{9} = 3$

$m_{sec} = \dfrac{f(b) - f(a)}{b - a} = \dfrac{3-1}{9-1} = \dfrac{1}{4}$

$y - 1 = \frac{1}{4}(x - 1)$

c. Find and graph any tangent lines to the graph of f that are parallel to the secant line.

$f(x) = x^{1/2}$

$f'(x) = \frac{1}{2}x^{-1/2} = \dfrac{1}{2\sqrt{x}}$

$\dfrac{1}{2\sqrt{x}} = \dfrac{1}{4}$

$2\sqrt{x} = 4$

$\sqrt{x} = 2$

$x = 4$