$$\frac{d}{dx}y = \frac{d}{dx}(x^3+8)^{10}$$

$$\frac{dy}{dx} = 10(x^3+8)^9 \cdot \frac{d}{dx}(x^3+8)$$

$$\frac{dy}{dx} = 10(x^3+8)^9 \cdot (3x^2)$$

$$dx\,\frac{dy}{dx} = 10(x^3+8)^9(3x^2)\,dx$$

$$\int dy = \int 10(x^3+8)^9(3x^2)\,dx$$

$$y = \frac{10(x^3+8)^{10}}{10} + C$$

consider $f(x) = 10x^9$, $g(x) = x^3+8$, $f(g(x)) = 10(x^3+8)^9$

$$\boxed{\int f(g(x))g'(x)\,dx = F(g(x)) + C}$$

or, you can let $g(x) = u$

$$\int f(u)\cdot du$$

pattern recog. $\int x\sqrt{1-x^2}\,dx$

$g(x) = 1-x^2$ $\quad f(x) = (\quad)^{1/2}$

$g'(x) = -2x$ $\quad F(x) = \frac{(\quad)^{3/2}}{3/2}$

$$\int x(1-x^2)^{1/2}dx$$

$$= -\frac{1}{2}\int -2x(1-x^2)^{1/2}dx$$

$$= -\frac{1}{2}\cdot F(g(x)) + C$$

$$= -\frac{1}{2}\cdot\frac{(1-x^2)^{3/2}}{\frac{3}{2}} + C$$

$$= \boxed{-\frac{1}{3}(1-x^2)^{3/2} + C}$$

change of variables $\int x\sqrt{1-x^2}\,dx$

Let $u = 1-x^2$

then

$$\frac{d}{dx}u = \frac{d}{dx}(1-x^2)$$

$$\frac{du}{dx} = -2x$$

$$dx = \frac{du}{-2x}$$

$$= \int \cancel{x}\cdot u^{1/2}\cdot\frac{du}{-2\cancel{x}}$$

$$= -\frac{1}{2}\int u^{1/2}du$$

$$= -\frac{1}{2}\cdot\frac{u^{3/2}}{3/2} + C$$

$$= -\frac{1}{3}u^{3/2} + C$$

$$= \boxed{-\frac{1}{3}(1-x^2)^{3/2} + C}$$

$$\int \cos 2x\,dx = \frac{1}{2}\int (\cos 2x)\,2\,dx$$

$f(x) = \cos x$

$F(x) = \sin x$

$g(x) = 2x$

$g'(x) = 2$

$$= \frac{1}{2}\cdot F(g(x)) + C$$

$$= \boxed{\frac{1}{2}\sin 2x + C}$$

$$\int \cos 2x\,dx = \int (\cos u)\frac{du}{2}$$

Let $u = 2x$

$\frac{du}{dx} = 2$

$dx = \frac{du}{2}$

$$= \frac{1}{2}\int \cos u\,du$$

$$= \frac{1}{2}\sin u + C$$

$$= \boxed{\frac{1}{2}\sin 2x + C}$$

$$\int \frac{x^4 - 1}{(x^5 - 5x + 2)^3}\,dx = \int (x^4 - 1)(x^5 - 5x + 2)^{-3}\,dx$$

Let $u = x^5 - 5x + 2$

$\frac{du}{dx} = 5x^4 - 5$

$dx = \frac{du}{5(x^4 - 1)}$

$$= \int \overset{1}{\cancel{(x^4 - 1)}}\, u^{-3} \cdot \frac{du}{5\underset{1}{\cancel{(x^4 - 1)}}}$$

$$= \frac{1}{5}\int u^{-3}\,du$$

$$= \frac{1}{5}\cdot\frac{u^{-2}}{-2} + C$$

$$= -\frac{1}{10}u^{-2} + C$$

$$= \boxed{-\frac{1}{10}(x^5 - 5x + 2)^{-2} + C}$$

Memorize:

$$\cos^2 A = \frac{1 + \cos 2A}{2}$$

$$\sin^2 A = \frac{1 - \cos 2A}{2}$$

your author isolates du

I isolate dx

4/14/15 (4.5 cont.)

Find the indefinite integral.

1) $\int 5\cos^2 2\theta \, d\theta$

$= 5\int \frac{1+\cos(2\cdot 2\theta)}{2} d\theta$

$= \frac{5}{2}\left[\int 1 \, d\theta + \int \cos 4\theta \, d\theta\right]$

$u = 4\theta$

$\frac{du}{d\theta} = 4$

$d\theta = \frac{du}{4}$

$= \frac{5}{2}\left[\theta + \int \cos u \frac{du}{4}\right]$

$= \frac{5}{2}\left[\theta + \frac{1}{4}\int \cos u \, du\right]$

$= \frac{5}{2}\left[\theta + \frac{1}{4}\sin u\right] + C$

$= \boxed{\frac{5}{2}\theta + \frac{5}{8}\sin 4\theta + C}$

2) $\int 3\cos 5x \sin^2 5x \, dx$

$u = \sin 5x$

$\frac{du}{dx} = 5\cos 5x$

$dx = \frac{du}{5\cos 5x}$

$= 3\int \cos 5x (\sin 5x)^2 \, dx$

$= 3\int (\cancel{\cos 5x}) u^2 \cdot \frac{du}{5\cancel{\cos 5x}}$

$= \frac{3}{5}\int u^2 \, du$

$= \frac{\cancel{3}}{5} \cdot \frac{u^3}{\cancel{3}} + C$

$= \boxed{\frac{1}{5}\sin^3 5x + C}$

$$\int_a^b f(g(x))g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du = F(u)\Big|_{g(a)}^{g(b)} = F(g(b)) - F(g(a))$$

Let $u = g(x)$, $\frac{du}{dx} = g'(x) \rightarrow du = g'(x)\,dx$

upper limit of int: $u = g(x) \rightarrow u = g(b)$

lower limit of int: $u = g(x) \rightarrow u = g(a)$

* Changing the limits of integration is the most proper way to do these problems.

* A less proper alternative is to consider the indefinite integral, find the antiderivative, back-substitute the original variables into the antiderivative, and then evaluate using the original limits of integration.

Evaluate the definite integral.

(1) $\int_0^{\pi/6} x\cos x^2\,dx$

Proper way:

$\int_0^{\pi/6} x\cos x^2\,dx = \int_0^{\pi^2/36} \cancel{x}\cos u\,\frac{du}{2\cancel{x}}$

$= \frac{1}{2}\int_0^{\pi^2/36}\cos u\,du$

$= \frac{1}{2}\sin u\Big|_0^{\pi^2/36}$

$= \frac{1}{2}\left(\sin\frac{\pi^2}{36} - \sin 0\right)$

$= \boxed{\frac{1}{2}\sin\frac{\pi^2}{36}}$

$u = x^2$

$\frac{du}{dx} = 2x$

$dx = \frac{du}{2x}$

upper limit: $u = x^2 \rightarrow u = \left(\frac{\pi}{6}\right)^2 = \frac{\pi^2}{36}$

lower limit: $u = x^2 \rightarrow u = (0)^2 = 0$

Less Proper Way

$\int_0^{\pi/6} x\cos x^2\,dx$

$= \frac{1}{2}\sin x^2\Big|_0^{\pi/6}$

$= \frac{1}{2}\sin\left(\frac{\pi}{6}\right)^2 - \frac{1}{2}\sin(0)^2 = \frac{1}{2}\sin\frac{\pi^2}{36}$

Consider $\int x\cos x^2\,dx = \int \cancel{x}\cos u\,\frac{du}{2\cancel{x}}$

$u = x^2$

$\frac{du}{dx} = 2x$

$dx = \frac{du}{2x}$

side work

$= \frac{1}{2}\int\cos u\,du$

$= \frac{1}{2}\sin u$

$= \frac{1}{2}\sin x^2$

MATH 150/GRACEY NAME______________________________

WORKSHEET/4.5-~~4.6~~

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1. Evaluate the following definite and indefinite integrals.

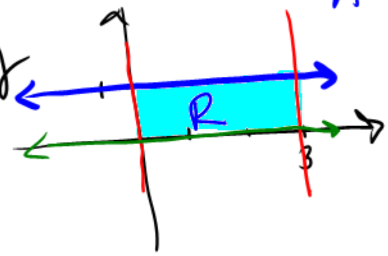
basic integral

a. $\int_0^3 dx = x\Big|_0^3$ OR

$= (3-0)$

$= \boxed{3}$

use geometry



$A = (3-0)(1) = 3$

---

b. $\int (3\csc\theta\cot\theta)\,d\theta$

no trick needed — basic integral

$= 3(-\csc\theta) + C$

$= \boxed{-3\csc\theta + C}$

---

used u-sub for binomial under radical, then wrote remaining x in terms of u and multiplied.

c. $\int \left(x\sqrt{1-x}\right)dx = \int x(1-x)^{1/2}\,dx$

$u = 1-x \rightarrow x = 1-u$

$\frac{du}{dx} = -1$

$dx = -du$

$= \int x \cdot u^{1/2}(-du)$

$= -\int (1-u)\cdot u^{1/2}\,du$

same trick

$= -\int (u^{1/2} - u^{3/2})\,du$

$= -\left(\frac{2}{3}u^{3/2} - \frac{2}{5}u^{5/2}\right) + C$

$= \boxed{-\frac{2}{3}(1-x)^{3/2} + \frac{2}{5}(1-x)^{5/2} + C}$

---

d. $\int x\left(5-2x^2\right)^5 dx$

$u = 5-2x^2$

$\frac{du}{dx} = -4x$

$dx = \frac{du}{-4x}$

u-sub

$= \int \cancel{x}\cdot u^5 \cdot \frac{du}{-4\cancel{x}}$

$= -\frac{1}{4}\int u^5\,du$

$= -\frac{1}{4}\cdot\frac{u^6}{6} + C$

$= \boxed{-\frac{1}{24}(5-2x^2)^6 + C}$

used usub for binomial under the cube root, wrote x in terms of u, then divided thru

$x = u-5$

$u = x+5$

$\frac{du}{dx} = 1$

$dx = du$

$\int \frac{x}{\sqrt[3]{x+5}}\,dx = \int \frac{x}{u^{1/3}}\,du$

$= \int \frac{u-5}{u^{1/3}}\,du$

$= \int \left(u^{2/3} - 5u^{-1/3}\right)du$

$= \frac{3}{5}u^{5/3} - 5\cdot\frac{3}{2}u^{2/3} + C$

$= \boxed{\frac{3}{5}(x+5)^{5/3} - \frac{15}{2}(x+5)^{2/3} + C}$

used the power-reducing identity, then split integral, used a u-sub on the 2nd integral

e. $\int \cos^2 3x\,dx = \int \frac{1+\cos 6x}{2}\,dx$

$u = 6x$

$\frac{du}{dx} = 6$

$dx = \frac{du}{6}$

$= \frac{1}{2}\left[\int 1\,dx + \int \cos 6x\,dx\right]$

$= \frac{1}{2}\left[x + \int \cos u \frac{du}{6}\right]$

$= \frac{1}{2}\left[x + \frac{1}{6}\int \cos u\,du\right] = \frac{1}{2}\left[x + \frac{1}{6}\sin u\right] + C$

$= \boxed{\frac{1}{2}x + \frac{1}{12}\sin 6x + C}$

no quotient rule!!! divide through by $x^{1/2}$

f. $\int \left(\frac{4+5x^{3/2}}{\sqrt{x}}\right)dx = \int (4x^{-1/2} + 5x)\,dx$

$= 4 \cdot 2x^{1/2} + 5\frac{x^2}{2} + C$

$= \boxed{8x^{1/2} + \frac{5}{2}x^2 + C}$

no quotient rule for integrals! Factor or use long or synthetic division

g. $\int_3^5 \frac{5+6x+x^2}{5+x}\,dx = \int_3^5 (x+1)\,dx$

$= \left(\frac{x^2}{2} + x\right)\Big|_3^5$

$= \left(\frac{25}{2} + 5\right) - \left(\frac{9}{2} + 3\right)$

$= \frac{16}{2} + 2$

$= \boxed{10}$

$\frac{x^2 + 6x + 5}{x+5} = 1x + 1 + \frac{0}{x+5} = x + 1$

divisor is $x - c$

$$\begin{array}{r|rrr} -5 & 1 & 6 & 5 \\ & & -5 & -5 \\ \hline & 1 & 1 & 0 \end{array}$$

$$|x-1| = \begin{cases} -(x-1), & x<1 \\ x-1 & x\ge 1\end{cases}$$

note: $\int_0^2 |x-1|\,dx = \left|2\int_0^1 (x-1)\,dx\right|$

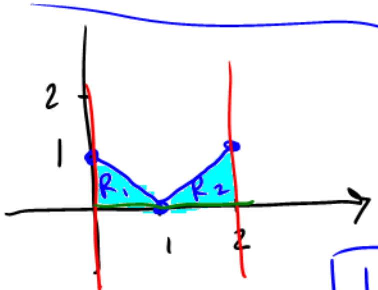
h. $\int_0^2 |x-1|\,dx = \int_0^1 -(x-1)\,dx + \int_1^2 (x-1)\,dx$

$$= -\left(\frac{x^2}{2} - x\right)\Big|_0^1 + \left(\frac{x^2}{2}-x\right)\Big|_1^2$$

$$= -\left[\left(\frac12 - 1\right) - (0-0)\right] + \left[\left(\frac42 - 2\right) - \left(\frac12 - 1\right)\right]$$

$$= \frac12 + 0 + \frac12$$

$$= \boxed{1}$$



$R_1 = \frac12\cdot 1\cdot 1$

$+R_2 = \frac12\cdot 1\cdot 1$

$\boxed{1}$

i. $\int_{\pi/4}^{\pi/3} \tan^3 x \sec^2 x\,dx = \int_1^{\sqrt3} u^3 \cancel{\sec^2 x}\cdot \frac{du}{\cancel{\sec^2 x}}$

$$= \int_1^{\sqrt3} u^3\,du$$

$$= \frac{u^4}{4}\Big|_1^{\sqrt3}$$

$$= \frac14\left[\left(3^{1/2}\right)^4 - (1)^4\right]$$

$$= \frac14(9-1)$$

$$= \boxed{2}$$

$u = \tan x$

$\frac{du}{dx} = \sec^2 x$

$dx = \frac{du}{\sec^2 x}$

upper: $u = \tan x \rightarrow u = \tan \pi/3 = \sqrt3$

lower: $u = \tan x \rightarrow u = \tan \pi/4 = 1$

2. Find the area of the region bounded by $y=(x-1)^2+1$, the $x-\text{axis}$, $x=-1$ and $x=2$.

$$A = \int_{-1}^{2} \left[(x-1)^2+1\right] dx$$

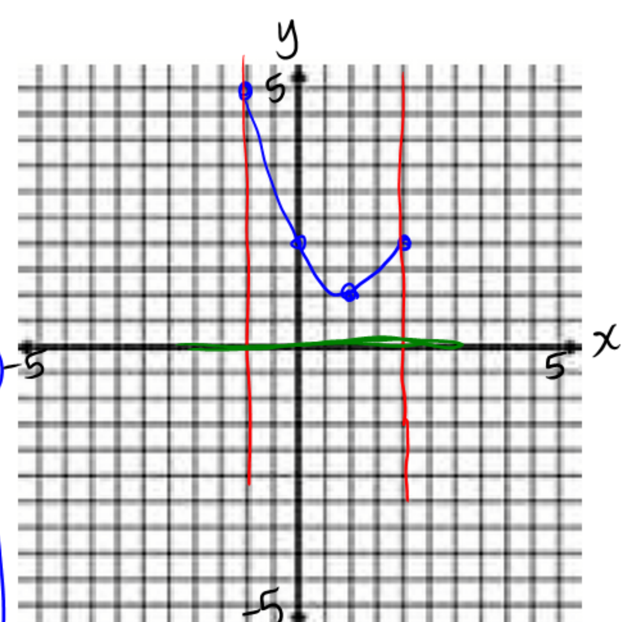
$$A = \int_{-2}^{1} u^2\,du + \int_{-1}^{2} 1\,dx$$

$$A = \frac{u^3}{3}\Big|_{-2}^{1} + x\Big|_{-1}^{2}$$

$$A = \left(\frac13 - \frac{-8}{3}\right) + (2-(-1))$$

$$A = \frac93 + 3$$

$$\boxed{A = 6 \text{ sq. units}}$$



$u = x-1$

$\frac{du}{dx} = 1$

$dx = du$

upper: $2-1=1$

lower: $-1-1=-2$