When you are done with your homework you should be able to…

- π Find the slope of the tangent line to a curve at a point
- π Use the limit definition to find the derivative of a function
- π Understand the relationship between differentiability and continuity

Warm-up: Find the following limits.

D.S. $\frac{0}{0}$

1. $\lim_{x\to 0}\frac{3x}{x^2+2x}$ $= \lim_{x\to 0}\frac{3\cancel{x}}{\cancel{x}(x+2)} \to$ D.S $= \frac{3}{0+2}$ $= \boxed{\frac{3}{2}}$

D.S. $\frac{0}{0}$

2. $\lim_{x\to 0}\frac{\frac{4}{4}\cdot\frac{1}{x+4}-\frac{1}{4}\cdot\frac{x+4}{x+4}}{x}$

$= \lim_{x\to 0}\frac{4-x-4}{4x(x+4)}$

$= \lim_{x\to 0}\frac{-\cancel{x}}{4\cancel{x}(x+4)}$

D.S. $= \frac{-1}{4(0+4)}$

$= \boxed{-\frac{1}{16}}$

D.S. $\frac{0}{0}$

3. $\lim_{\Delta x\to 0}\frac{(x+\Delta x)^2-x^2}{\Delta x}$ $= \lim_{\Delta x\to 0}\frac{[(x+\Delta x)+x][(x+\Delta x)-x]}{\Delta x}$

$= \lim_{\Delta x\to 0}\frac{(2x+\Delta x)(\cancel{\Delta x})}{\cancel{\Delta x}}$

D.S. $= 2x+0$

$= \boxed{2x}$

you just evaluated the derivate w/respect to x of $f(x)=x^2$

DEFINITION OF TANGENT LINE WITH SLOPE *m*

If f is defined on an open interval containing *c*, and if the limit

$$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{f(c+\Delta x)-f(c)}{\Delta x}=m$$

exists, then the line passing through $(c,f(c))$ with slope m is the **tangent line** to the graph of f at the point $(c,f(c))$.

The slope of the tangent line to the graph of f at the point $(c,f(c))$ is also called the **slope of the graph of *f* at *x* = *c*.

Example 1: Find the slope of the graph of $f(x)=6-x^2$ at the point $(1,5)$.

$$m=\lim_{\Delta x\to 0}\frac{f(c+\Delta x)-f(c)}{\Delta x}$$

$$m=\lim_{\Delta x\to 0}\frac{6-(1+\Delta x)^2-5}{\Delta x}$$

$$m=\lim_{\Delta x\to 0}\frac{1^2-(1+\Delta x)^2}{\Delta x}$$

$$m=\lim_{\Delta x\to 0}\frac{[1+(1+\Delta x)][1-(1+\Delta x)]}{\Delta x}$$

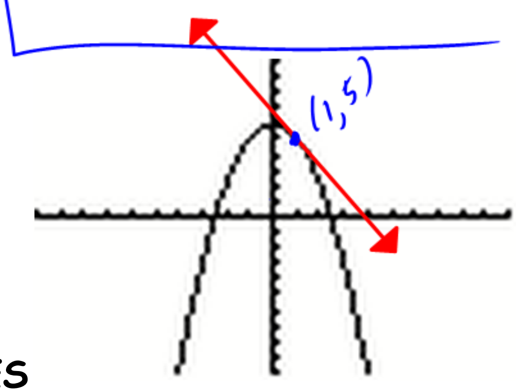
$$m=\lim_{\Delta x\to 0}\frac{(2+\Delta x)(-\cancel{\Delta x})}{\cancel{\Delta x}}$$

$$m\overset{D.S}{=}-(2+0)$$

$$\boxed{m=-2}$$

$f(c)=f(1)=5$

$$\boxed{f(c+\Delta x)=f(1+\Delta x)=6-(1+\Delta x)^2}$$



DEFINITION FOR VERTICAL TANGENT LINES

If f is continuous at *c* and

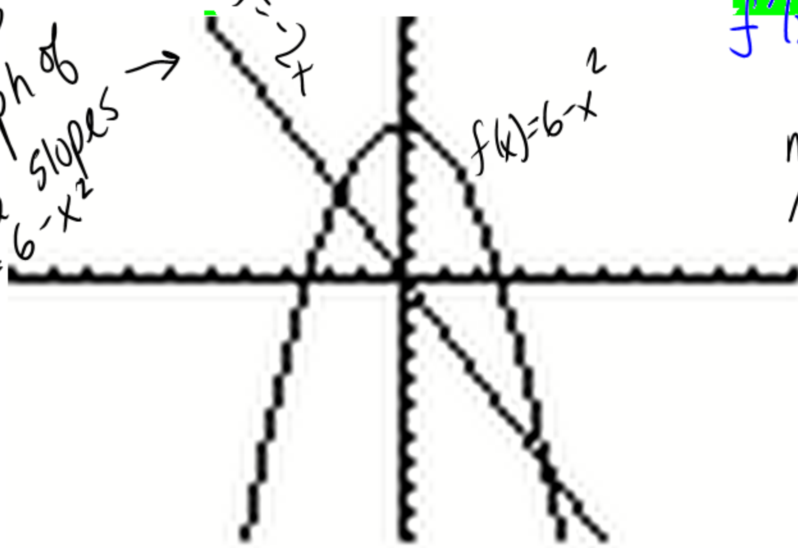
$$\lim_{\Delta x\to 0}\frac{f(c+\Delta x)-f(c)}{\Delta x}=\infty \quad\text{or}\quad \lim_{\Delta x\to 0}\frac{f(c+\Delta x)-f(c)}{\Delta x}=-\infty$$

the vertical line $x=c$ passing through the point $(c,f(c))$ is a **vertical tangent line** to the graph of f.

$f(x) = 6 - x^2$

$f'(x) = -2x$

This is the graph of all possible slopes of $f(x) = 6 - x^2$ →



nderiv(
TI-83

$\frac{dy}{dx}$(
TI-84

DEFINITION OF THE DERIVATIVE OF A FUNCTION

The **derivative** of f at x is given by

$$f'(x)=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$$

provided the limit exists. For all x for which this limit exists, f' is a function of x.

The process of finding the derivative of a function is called

differentiation.

A function is diffentiable at x if its derivative exists at x and is differentiable on an open interval (a, b) if it is differentiable at every point in the interval.

NOTATION FOR THE DERIVATIVE OF $y=f(x)$:

$f'(x)$, $y'(x)$, $\frac{d}{dx}(f(x))$, $\frac{dy}{dx}$, $D_x[f(x)]$

The derivative of f with respect to x.

Example 2: Find the derivative of $f(x)=4-x^3$ using the limit process.

$$f'(x)=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$$

$$f'(x)=\lim_{\Delta x\to 0}\frac{[4-(x+\Delta x)^3]-[4-x^3]}{\Delta x}$$

$$f'(x)=\lim_{\Delta x\to 0}\frac{\cancel{4}-\cancel{x^3}-3x^2\Delta x-3x\Delta x^2-\Delta x^3-\cancel{4}+\cancel{x^3}}{\Delta x}$$

Pascal's Δ

$(a+b)^0=1$

$(a+b)^1=1a+1b$

$(a+b)^2=1a^2b^0+2a^1b^1+1a^0b^2$

$(x+\Delta x)^3$

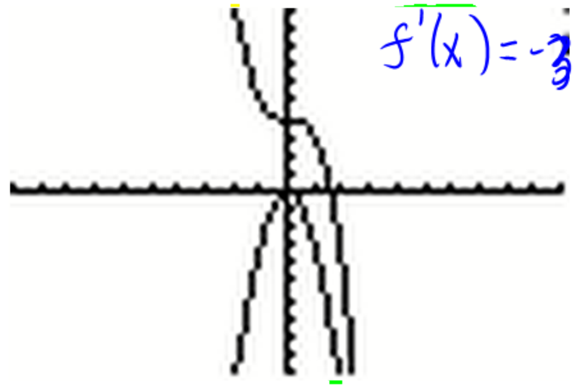
$=1x^3\Delta x^0+3x^2\Delta x^1+3x^1\Delta x^2+1x^0\Delta x^3$

$=x^3+3x^2\Delta x+3x\Delta x^2+\Delta x^3$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

$f(x) = 4 - x^3$

$f'(x) = -3x^2$



$$f'(x) = \lim_{\Delta x \to 0} \frac{\cancel{\Delta x}\left(-3x^2 - 3x\Delta x - \Delta x^2\right)}{\cancel{\Delta x}}$$

$$f'(x) = \lim_{\Delta x \to 0} \left(-3x^2 - 3x\Delta x - \Delta x^2\right)$$

$$f'(x) \overset{D.S.}{=} -3x^2 - 3x(0) - (0)^2$$

$$\boxed{f'(x) = -3x^2}$$

ALTERNATIVE LIMIT FORM OF THE DERIVATIVE

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

This form of the derivative requires that the one-sided limits

$\lim_{x \to c^-} \frac{f(x) - f(c)}{x - c}$ and $\lim_{x \to c^+} \frac{f(x) - f(c)}{x - c}$ exist and are equal.

Example 3: Is the function $f(x) = x^{2/3}$ differentiable at $x = 0$?

$f(x) = x^{2/3}$

$c = 0$

$f(c) = 0$

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

$$\lim_{x \to 0^+} \frac{x^{2/3} - 0}{x - 0} = \lim_{x \to 0^+} \frac{x^{2/3}}{x}$$

$$= \lim_{x \to 0^+} \frac{1}{x^{1/3}}$$

$$= \infty$$

DNE

f is not differentiable at $x = 0$

THEOREM: DIFFERENTIABILITY IMPLIES CONTINUITY

If f is differentiable at $x = c$, then f is continuous at $x = c$.