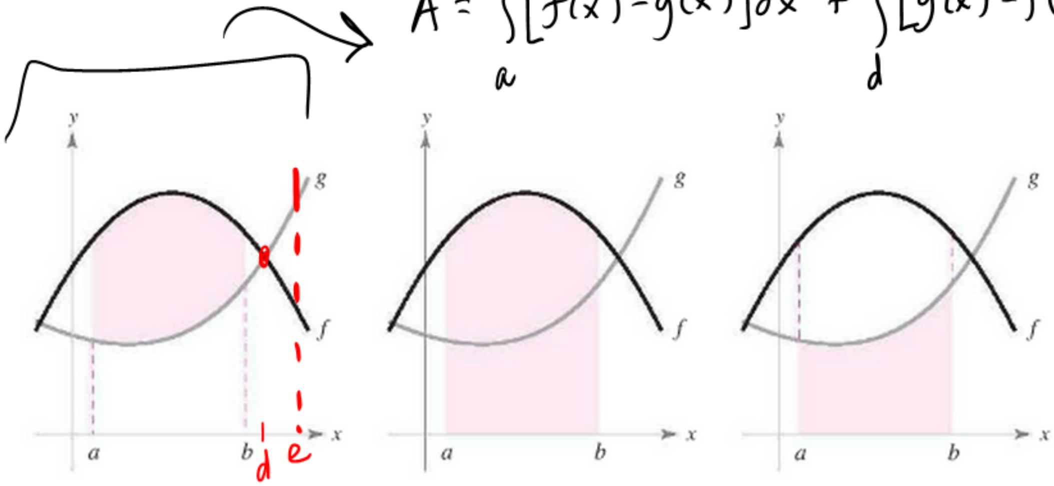
7.1: Area Between Curves

$$A = \int_a^d [f(x)-g(x)]\,dx + \int_d^e [g(x)-f(x)]\,dx$$



Area of region between f and g	=	Area of region under f	−	Area of region under g
$\int_a^b [f(x) - g(x)]\,dx$	=	$\int_a^b f(x)\,dx$	−	$\int_a^b g(x)\,dx$

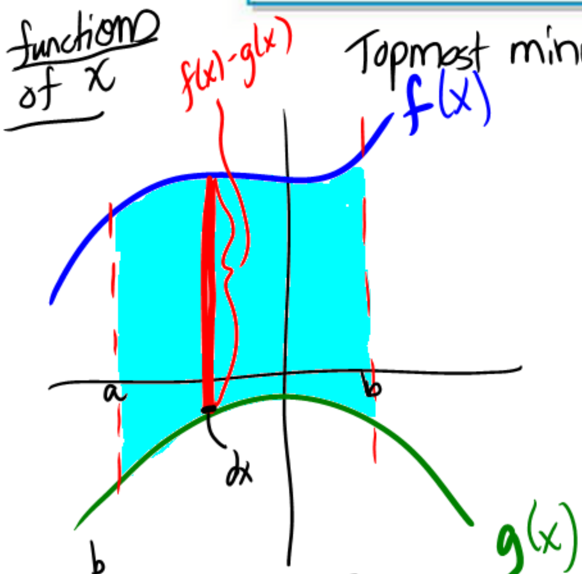
AREA OF A REGION BETWEEN TWO CURVES

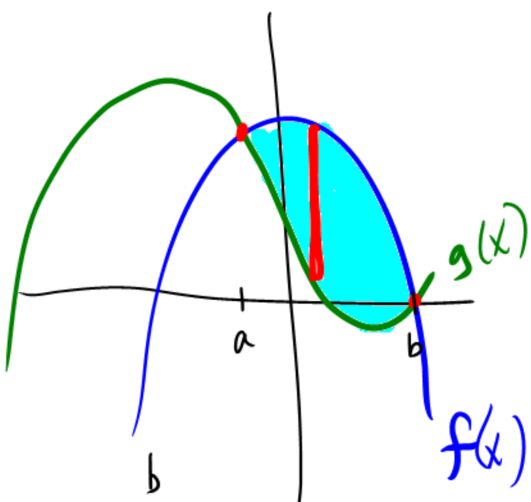
If f and g are continuous on $[a, b]$ and $g(x) \leq f(x)$ for all x in $[a, b]$, then the area of the region bounded by the graphs of f and g and the vertical lines $x = a$ and $x = b$ is

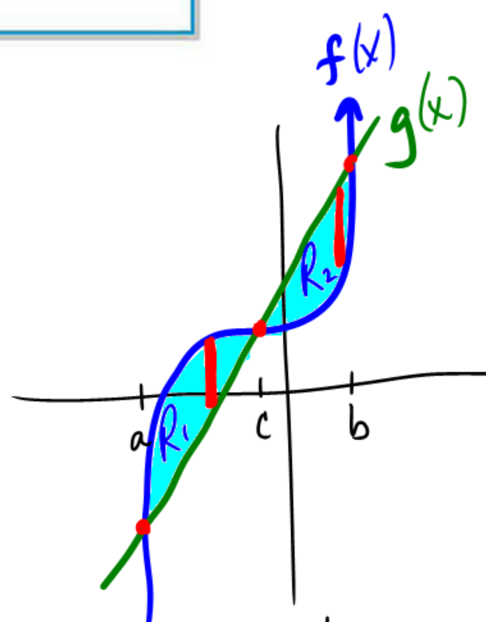
$$A = \int_a^b [f(x) - g(x)]\,dx.$$

functions of x

Topmost minus bottom most







$$A = \int_a^b [f(x)-g(x)]\,dx$$

a and b are provided in the form of vertical lines or an interval about x

$$A = \int_a^b [f(x)-g(x)]\,dx$$

no bounds provided so equate f and g to find limits of integ.

$$A = \int_a^c [f(x)-g(x)]\,dx + \int_c^b [g(x)-f(x)]\,dx$$

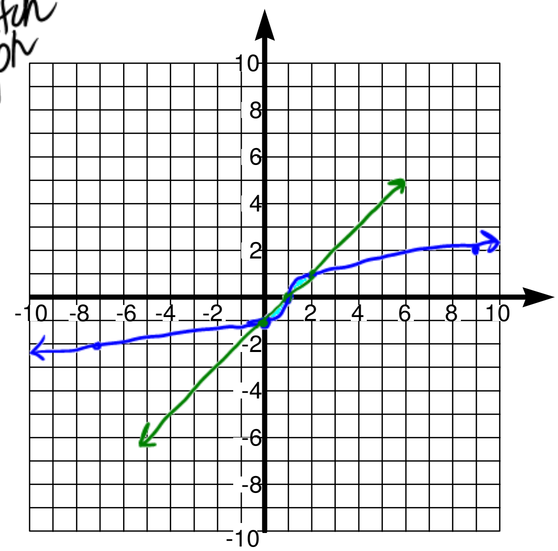
no bounds provided so equate f and g to find limits of integ.

b. $\cancel{\dfrac{dr}{ds} = e^{r-2s}}$

$\cancel{r(0) = 0}$ SKIP

7. Sketch the region bounded by the graphs of the algebraic functions and find the area of the region.

a. $f(x) = \sqrt[3]{x-1},\ g(x) = x-1$ ① Sketch graph



② Find limits of integration

$$\left(\sqrt[3]{x-1}\right)^3 = (x-1)^3$$

$$x-1 = (x-1)^3$$

$$0 = (x-1)^3 - (x-1)$$

$$0 = (x-1)\left[(x-1)^2 - 1\right]$$

$0 = x-1$ $\quad 0 = (x-1)^2 - 1$

$x = 1$ $\quad \sqrt{1} = \sqrt{(x-1)^2}$

$$\pm 1 = x-1$$

$$x = 0, +2$$

③ Find Area

$$A = \int_0^1 \left[(x-1) - \sqrt[3]{x-1}\right]dx + \int_1^2 \left[\sqrt[3]{x-1} - (x-1)\right]dx$$

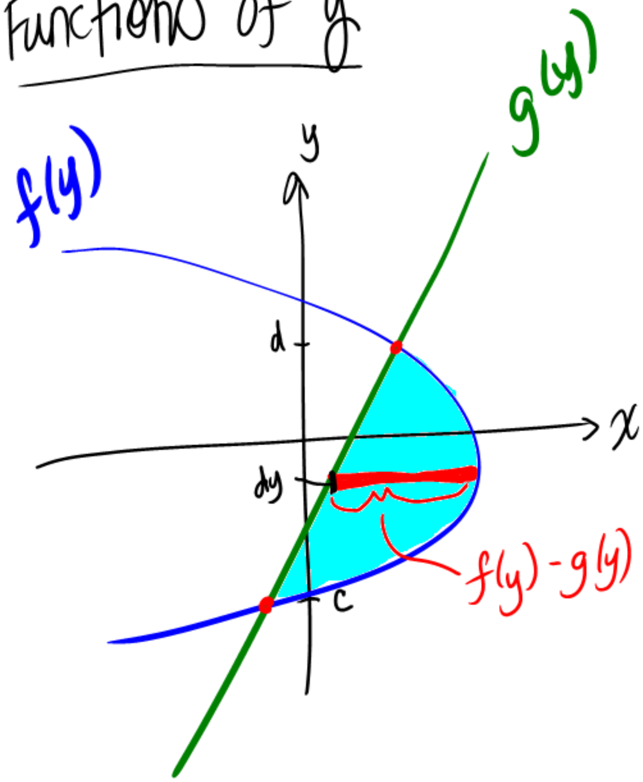
$$A = \left[\frac{(x-1)^2}{2} - \frac{3}{4}(x-1)^{4/3}\right|_0^1 + \left[\frac{3}{4}(x-1)^{4/3} - \frac{(x-1)^2}{2}\right|_1^2$$

$$A = \left[(0-0) - \left(\tfrac{1}{2} - \tfrac{3}{4}(1)\right)\right] + \left[\left(\tfrac{3}{4}(1) - \tfrac{1}{2}\right) - (0-0)\right]$$

$$A = \frac{1}{4} + \frac{1}{4} \rightarrow \boxed{A = \tfrac{1}{2} \text{ sq. unit}}$$

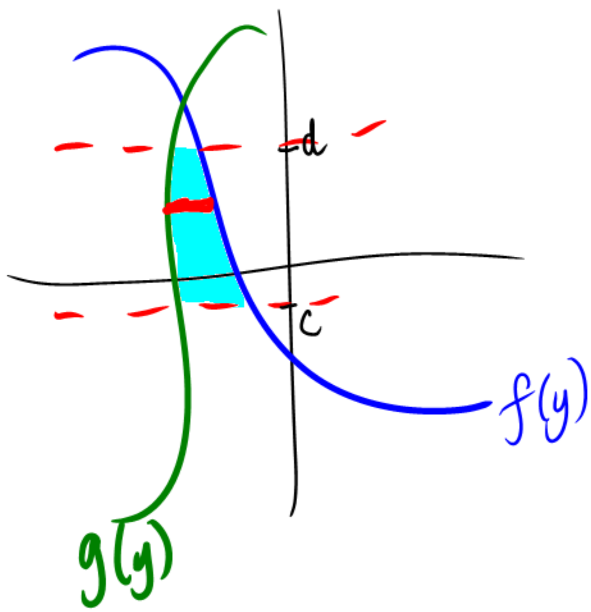
rightmost minus leftmost

Functions of y



no bounds provided
so equate f and g
to find limits of integ.

$$A = \int_c^d [f(y) - g(y)]\,dy$$



Bounds are given in the form of horizontal lines or an interval about y.

$$A = \int_c^d [f(y) - g(y)]\,dy$$

$x = y^2 \quad x = y+2$

b. $f(y) = y^2,\ g(y) = y + 2$

① Sketch graph

② Find limits of int.

$y^2 = y+2$

$0 = y^2 - y - 2$

$0 = (y-2)(y+1)$

$y = 2$ or $y = -1$

③ Find Area

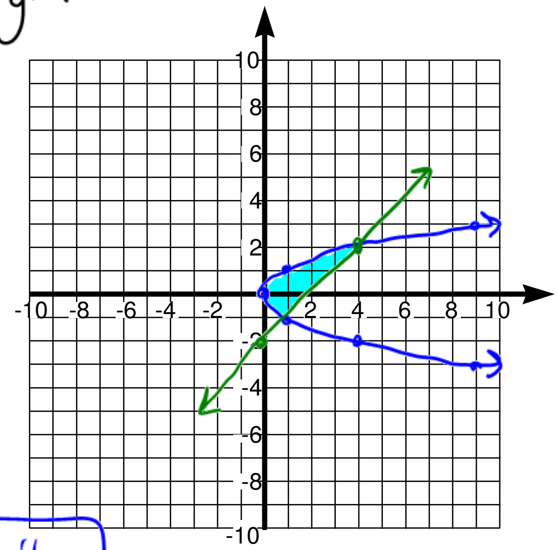
$A = \int_{-1}^{2} [(y+2) - y^2]\, dy$

$A = \left(\frac{(y+2)^2}{2} - \frac{y^3}{3}\right)\Big|_{-1}^{2}$

$A = \left(\frac{16}{2} - \frac{8}{3}\right) - \left(\frac{1}{2} - \frac{-1}{3}\right)$

$A = 8 - 3 - \frac{1}{2}$

$A = \frac{10}{2} - \frac{1}{2} \rightarrow \boxed{A = \frac{9}{2} \text{ sq. units}}$



8. Set up and evaluate the definite integral that gives the area of the region bounded by the graph of the function and the tangent line to the graph at the given point.

$y_1 = x(x^2 - 2)$

$y = x^3 - 2x,\ (-1, 1)$, $y'(x) = 3x^2 - 2 \rightarrow y'(-1) = 1$

$y_1 = x^3 - 2x$ and $y_2 = x + 2$

Limits of integration:

$x^3 - 2x = x + 2$

$x^3 - 3x - 2 = 0$

$$\begin{array}{r|rrrr} -1 & 1 & 0 & -3 & -2 \\ & & -1 & 1 & 2 \\ \hline & 1 & -1 & -2 & 0 \end{array}$$

$(x+1)(x^2 - x - 2) = 0$

$(x+1)(x+1)(x-2) = 0$

$x = -1$ (mult 2), $x = 2$

Area:

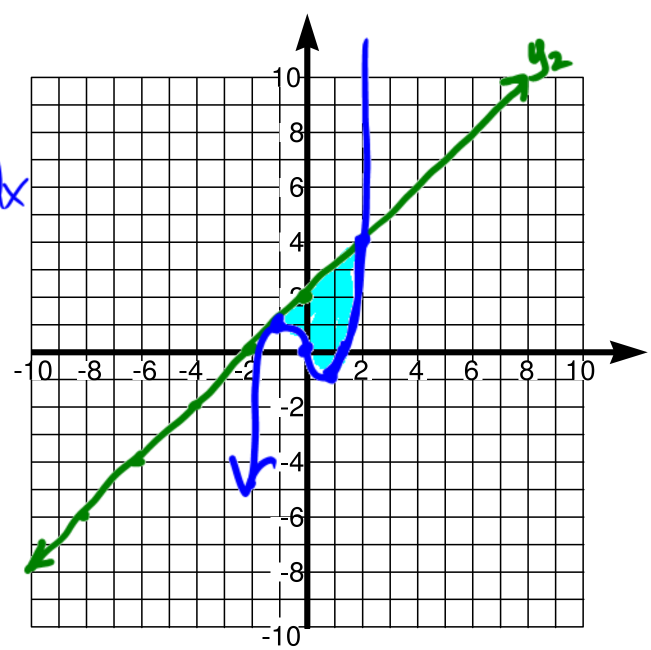
$A = \int_{-1}^{2} [(x+2) - (x^3 - 2x)]\, dx$

$A = \left(\frac{(x+2)^2}{2} - \frac{x^4}{4} + x^2\right)\Big|_{-1}^{2}$

$A = (8 - 4 + 4) - \left(\frac{1}{2} - \frac{1}{4} + 1\right)$

$A = 7 - \frac{1}{4}$

$\boxed{A = \frac{27}{4} \text{ sq. units}}$



$* y - 1 = 1(x+1)$

$y = x + 2$