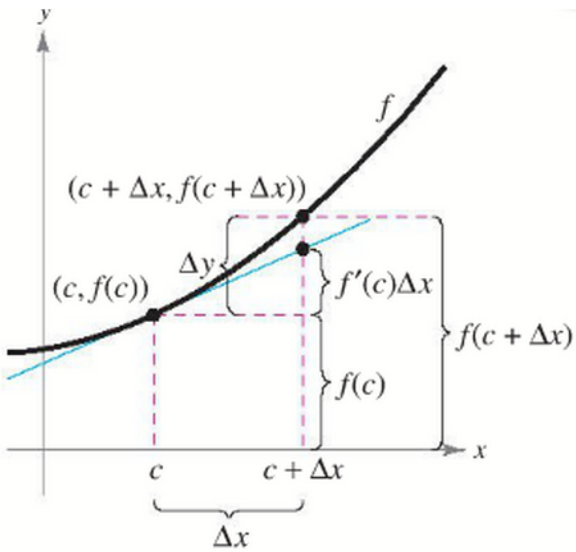
3.9: Differentials



When Δx is small, $\Delta y = f(c + \Delta x) - f(c)$ is approximated by $f'(c)\Delta x$.

Definition of Differentials

Let $y = f(x)$ represent a function that is differentiable on an open interval containing x. The **differential of x** (denoted by dx) is any nonzero real number. The **differential of y** (denoted by dy) is

$$dy = f'(x)\,dx.$$

$$\frac{dy}{dx} = f'(x) \longrightarrow dy = f'(x)\,dx$$

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u, v are functions of x.

1) $\frac{d}{dx}(uv) = u \cdot \frac{dv}{dx} + v \cdot \frac{du}{dx}$

$d(uv) = u\,dv + v \cdot du$

area of a triangular region

↳ propagated error

$A = \frac{1}{2}bh$

$dA = \frac{1}{2}[b\,dh + h\,db]$

Differential Formulas

Let u and v be differentiable functions of x.

Constant multiple: $d[cu] = c\,du$

Sum or difference: $d[u \pm v] = du \pm dv$

Product: $d[uv] = u\,dv + v\,du$

Quotient: $d\left[\frac{u}{v}\right] = \frac{v\,du - u\,dv}{v^2}$

dA : error in area

db : error in base

dh: error in height

27. Area The measurements of the base and altitude of a triangle are found to be 36 and 50 centimeters, respectively. The possible error in each measurement is 0.25 centimeter.

(a) Use differentials to approximate the possible propagated error in computing the area of the triangle.

(b) Approximate the percent error in computing the area of the triangle.

a) $A = \frac{1}{2}bh$

$b = 36\text{ cm}$

$h = 50\text{ cm}$

$db = \pm 0.25$

$dh = \pm 0.25$

want dA

$dA = \frac{1}{2}\left[b\,dh + h\,db\right]$

$dA = \frac{1}{2}\left[(36)(.25) + (50)(.25)\right]$

$dA = \frac{1}{2}\left[9 + 12.5\right]$

$dA = \pm 10.75\text{ cm}^2$

$\boxed{\text{The possible propagated error in the area is } \pm 10.75\text{cm}^2.}$

$f(x+\Delta x) - f(x) = \Delta y$

$f(x+\Delta x) - f(x) = f'(x)dx$

$$\boxed{f(x+\Delta x) = f(x) + f'(x)dx}$$

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82/27
        3.037037037
28^(1/3)
        3.036588972
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ex: Use differentials to approximate:

a) $\sqrt[3]{28}$

$x = 27,\ dx = 1$

$f(x) = \sqrt[3]{x},\ f'(x) = \frac{1}{3}x^{-2/3}$

$\sqrt[3]{27+1} = f(27) + f'(27)dx$

$= \sqrt[3]{27} + \frac{1}{3}(27)^{-2/3}\cdot(1)$

$= 3 + \frac{1}{3}\cdot\frac{1}{9}\cdot 1$

$= \frac{82}{27}$

b) $\sin(0.1)$

$x = 0,\ dx = 0.1$

$f(x) = \sin x,\ f'(x) = \cos x$

$\sin(0+0.1) = f(0) + f'(0)dx$

$= \sin 0 + (\cos 0)(.1)$

$= 0.1$