MATH 150/GRACEY NAME Key

EXAM 4/4.1-4.5 HW_______% QUIZ_______% EXAM_______%

100 POINTS POSSIBLE/BOX YOUR FINAL ANSWER

EXACT ANSWERS ONLY UNLESS OTHERWISE INDICATED

SHOW ALL WORK FOR FULL CREDIT!!!

(64 POINTS) Problems 1-8. Evaluate the definite integrals and find the indefinite integrals: Each question is worth 8 points. EXACT ANSWERS ONLY!!!

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1. $\int_{-2}^{4} |x-2|\,dx$

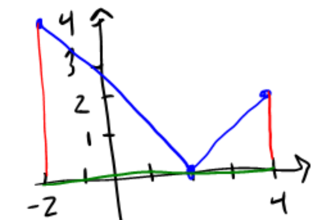
$$= -\int_{-2}^{2}(x-2)\,dx + \int_{2}^{4}(x-2)\,dx$$

$$= -\left(\frac{x^2}{2}-2x\right)\Big|_{-2}^{2} + \left(\frac{x^2}{2}-2x\right)\Big|_{2}^{4}$$

$$= -\left[\left(\frac{4}{2}-4\right)-\left(\frac{4}{2}+4\right)\right] + \left[\left(\frac{16}{2}-8\right)-\left(\frac{4}{2}-4\right)\right]$$

$$= -(-8) + (0-(-2))$$

$$= \boxed{10}$$



$A = \frac{1}{\cancel{2}}\cdot\cancel{4}^{2}\cdot 4 + \frac{1}{\cancel{2}}\cdot\cancel{2}\cdot 2$

$A = 10$

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2. $\int \frac{\sec\sqrt{\theta}\,\tan\sqrt{\theta}}{\sqrt{\theta}}\,d\theta$

$$= \int \frac{\sec u \tan u}{\cancel{\sqrt{\theta}}}\cdot 2\cancel{\sqrt{\theta}}\,du$$

$$= 2\int \sec u \tan u\,du$$

$$= 2\sec u + C$$

$$= \boxed{2\sec\sqrt{\theta} + C}$$

$u = \sqrt{\theta}$

$\frac{du}{d\theta} = \frac{1}{2\sqrt{\theta}}$

$d\theta = 2\sqrt{\theta}\,du$

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3. $\int \frac{x}{\sqrt{5-x}}\,dx$

$$= \int x(5-x)^{-1/2}\,dx$$

$$= \int \cancel{x}\cdot u^{-1/2}(-du)$$

$$= -\int (5-u)u^{-1/2}$$

$$= -\int \left(5u^{-1/2} - u^{1/2}\right)du$$

$$= -\left(5\frac{u^{1/2}}{1/2} - \frac{u^{3/2}}{3/2}\right) + C$$

$$= \boxed{-10(5-x)^{1/2} + \frac{2}{3}(5-x)^{3/2} + C}$$

$u = 5-x,\ x = 5-u$

$\frac{du}{dx} = -1$

$dx = -du$

4. $\int x^3 (x^4+1)^3 dx = \int \cancel{x^3} \cdot u^3 \frac{du}{4\cancel{x^3}}$

$u = x^4+1$

$\frac{du}{dx} = 4x^3$

$dx = \frac{du}{4x^3}$

$= \frac{1}{4}\int u^3 du$

$= \frac{1}{4} \cdot \frac{u^4}{4} + C$

$= \boxed{\frac{1}{16}(x^4+1)^4 + C}$

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5. $\int \sin^2 4x dx = \int \frac{1-\cos(2 \cdot 4x)}{2} dx$

$u = 8x$

$\frac{du}{dx} = 8$

$dx = \frac{du}{8}$

$= \frac{1}{2}\left[\int 1 dx - \int \cos 8x dx\right]$

$= \frac{1}{2}\left[x - \int \cos u \frac{du}{8}\right]$

$= \frac{1}{2}\left[x - \frac{1}{8}\int \cos u du\right]$

$= \frac{1}{2}\left[x - \frac{1}{8}(\sin u)\right] + C$

$= \boxed{\frac{1}{2}x - \frac{1}{16}\sin 8x + C}$

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6. $\int \left(\frac{x^2+\sqrt{x}}{\sqrt[3]{x}}\right) dx = \int (x^2 + x^{1/2}) x^{-1/3} dx$

$= \int (x^{5/3} + x^{1/6}) dx$

$= \frac{x^{8/3}}{8/3} + \frac{x^{7/6}}{7/6} + C$

$= \boxed{\frac{3}{8}x^{8/3} + \frac{6}{7}x^{7/6} + C}$

7. $\displaystyle\int_{\pi/6}^{\pi/3} \frac{1}{1-\sin x}\,dx$

$$= \int_{\pi/6}^{\pi/3} \frac{1}{(1-\sin x)}\frac{(1+\sin x)}{(1+\sin x)}\,dx$$

$$= \int_{\pi/6}^{\pi/3} \frac{1+\sin x}{1-\sin^2 x}\,dx$$

$$= \int_{\pi/6}^{\pi/3} \frac{1+\sin x}{\cos^2 x}\,dx$$

$$= \int_{\pi/6}^{\pi/3} \left(\frac{1}{\cos^2 x} + \frac{1}{\cos x}\cdot\frac{\sin x}{\cos x}\right)dx$$

$$= \int_{\pi/6}^{\pi/3} (\sec^2 x + \sec x \tan x)\,dx$$

$$= \left(\tan x + \sec x\right|_{\pi/6}^{\pi/3}$$

$$= \left[\left(\tan\tfrac{\pi}{3} + \sec\tfrac{\pi}{3}\right) - \left(\tan\tfrac{\pi}{6} + \sec\tfrac{\pi}{6}\right)\right]$$

$$= \sqrt{3} + 2 - \frac{\sqrt{3}}{3} - \frac{2\sqrt{3}}{3}$$

zero out $= \dfrac{\cancel{3\sqrt{3}} + 6 - \cancel{\sqrt{3}} - \cancel{2\sqrt{3}}}{3}$

$$= \boxed{2}$$

8. $\displaystyle\int \frac{\cos^2 x - \sin^2 x}{\cos 2x}\,dx$

$$= \int \frac{\cos 2x}{\cos 2x}\,dx$$

$$= \int 1\,dx$$

$$= \boxed{x + C}$$

9. (9 POINTS) Find the average value of the function $f(x) = x\sqrt{x^2+7}$ on the interval $[0,3]$.

Average value $= \dfrac{1}{b-a}\displaystyle\int_a^b f(x)\,dx$

$$= \frac{1}{3-0}\int_0^3 x(x^2+7)^{1/2}\,dx$$

$u = x^2+7$

$\dfrac{du}{dx} = 2x$

$dx = \dfrac{du}{2x}$

upper limit: $u = x^2+7 = 16$

lower limit: $u = x^2+7 = 7$

$$= \frac{1}{3}\int_7^{16} \cancel{x}\cdot u^{1/2}\,\frac{du}{2\cancel{x}}$$

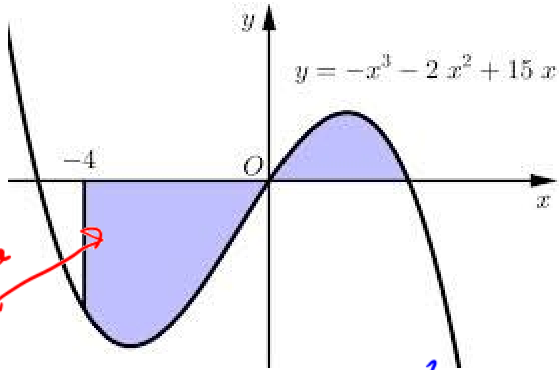
$$= \frac{1}{6}\int_7^{16} u^{1/2}\,du$$

$$= \frac{1}{6}\left(\frac{u^{3/2}}{3/2}\right|_7^{16}$$

$$= \frac{1}{9}\left((\sqrt{16})^3 - (\sqrt{7})^3\right)$$

$$= \boxed{\frac{1}{9}\left(64 - 7\sqrt{7}\right)}$$

10. (9 POINTS) Find the **area** of the shaded region. Show all work for full credit.



negative " area "

$0 = -x^3 - 2x^2 + 15x$

$0 = -x(x^2 + 2x - 15)$

$-x = 0$ or $x^2 + 2x - 15 = 0$

$x = 0$ $\quad (x+5)(x-3) = 0$

$x + 5 = 0$ or $x - 3 = 0$

$x = -5$ $\quad x = 3$

$A = -\int^{0} (-x^3 - 2x^2 + 15x)\,dx + \int_0^3 (-x^3 - 2x^2 + 15x)\,dx$

$A = -\left(-\frac{1}{4}x^4 - \frac{2}{3}x^3 + \frac{15}{2}x^2\right)\Big|_{-5}^{0} + \left(-\frac{1}{4}x^4 - \frac{2}{3}x^3 + \frac{15}{2}x^2\right)\Big|_0^3$

$A = -\left[(0-0+0) - \left(-\frac{256}{4} + \frac{128}{3} + \frac{240}{2}\right)\right] + \left[\left(-\frac{81}{4} - 18 + \frac{135}{2}\right) - (0-0+0)\right]$

$A = 64 + \frac{128}{3} + 120 - \frac{81}{4} - 18 + \frac{135}{2} \rightarrow A = 38 + \frac{-512+243+810}{12} \rightarrow A = \frac{456}{12} + \frac{1079}{12}$

$\rightarrow A = \frac{1535}{12}$ sq. units

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11. (8 POINTS) Sketch the region whose area is given by the definite integral. Then **use a geometric formula** to evaluate the integral. $\int_{-4}^{4} \sqrt{16 - x^2}\,dx$.

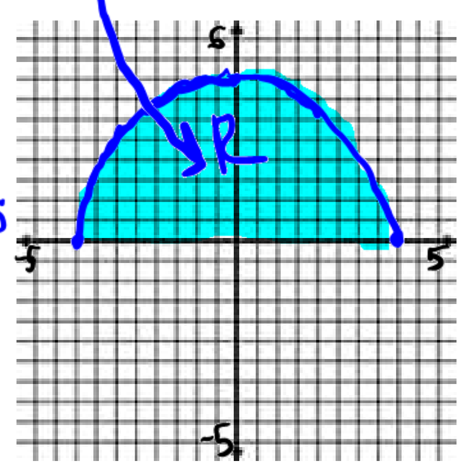
$\text{Area} = \frac{\pi r^2}{2}$

$\text{Area} = \frac{\pi (4)^2}{2}$

Area = 8π sq. units

$\boxed{\int_{-4}^{4} \sqrt{16 - x^2}\,dx = 8\pi}$

circle with radius of 4



12. (10 POINTS) Evaluate the definite integral by the **limit definition**.

$$\int_0^3 (x^2+2)\,dx = \lim_{\|\Delta\|\to 0} \sum_{i=1}^{n} f(c_i)\Delta x_i$$

$a=0,\ b=3$

$\Delta x_i = \Delta x = dx = \frac{b-a}{n} = \frac{3}{n}$

$c_i = a + i\Delta x_i$ ← regular partions

$c_i = a + i(\Delta x)$

$c_i = a + i(dx)$

$c_i = 0 + i\left(\frac{3}{n}\right)$

$c_i = 3i/n$

$f(c_i) = f\left(\frac{3i}{n}\right)$

$f(c_i) = \left(\frac{3i}{n}\right)^2 + 2$

$f(c_i) = \frac{9i^2}{n^2} + 2$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{9i^2}{n^2}+2\right)\left(\frac{3}{n}\right)$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{27i^2}{n^3} + \frac{6}{n}\right)$$

$$= \lim_{n\to\infty} \left[\frac{27}{n^3} \sum_{i=1}^{n} i^2 + \frac{1}{n}\sum_{i=1}^{n} 6\right]$$

$$= \lim_{n\to\infty} \left[\frac{\overset{9}{\cancel{27}}}{n^{\cancel{3}\,2}} \cdot \frac{\cancel{n}(n+1)(2n+1)}{\underset{2}{\cancel{6}}} + \frac{1}{\cancel{n}} \cdot 6\cancel{n}\right]$$

$$= \lim_{n\to\infty} \left(\frac{9(2n^2+3n+1)}{2n^2} + 6\right)$$

$$= \lim_{n\to\infty} \left(\frac{18n^2}{2n^2} + \frac{27n}{2n^2} + \frac{9}{2n^2} + 6\right)$$

$$= \lim_{n\to\infty} \left(9 + \frac{27}{n} + \frac{9}{2n^2} + 6\right)$$

$$= 9 + 0 + 0 + 6$$

$$= \boxed{15}$$

Theorem: Summation Formulas

1. $\sum_{i=1}^{n} c = cn$	2. $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$
3. $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$	4. $\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$