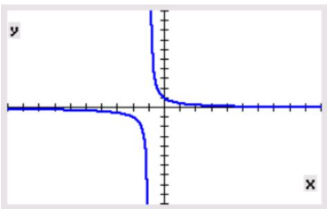
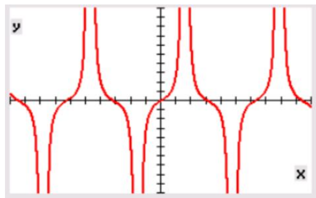
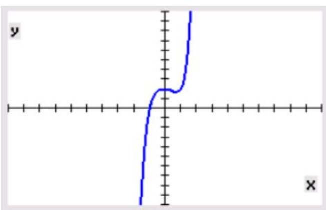
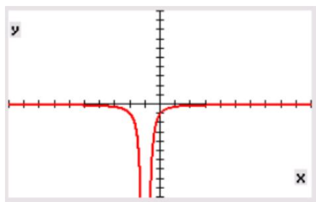
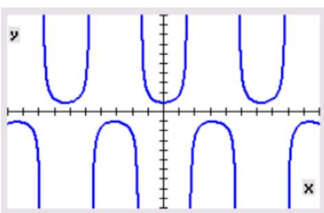
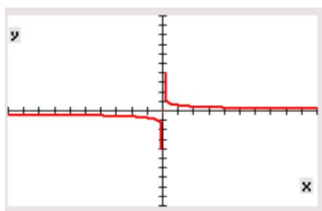
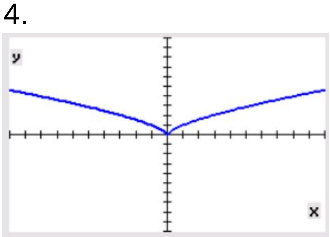
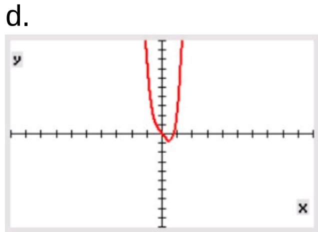
MATH 150/GRACEY NAME Key

EXAM 3/PART 2/50 POINTS POSSIBLE

GRAPHING CALCULATOR PERMITTED

(8 POINTS) Match the graph of f in the left column with that of its derivative in the right column.

GRAPH OF f	GRAPH OF f'
1. 	a. 
2. 	b. 
3. 	c. 
4. 	d. 

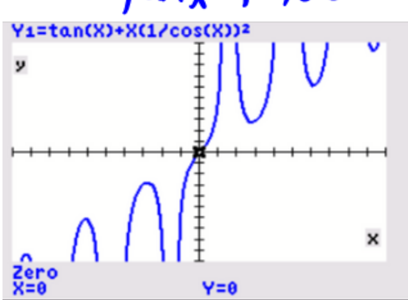
1. b
2. d
3. a
4. c

5. (8 points) **Use calculus** to find the absolute extreme values of the function and the ordered pairs where they occur. You may round to the nearest hundredth.

$f(x) = x\tan x$ on the interval $\left[\frac{\pi}{6}, \frac{\pi}{3}\right]$

$f'(x) = \tan x + x\sec^2 x$

$0 = \tan x + x\sec^2 x$



$x = 0 \notin [\pi/6, \pi/3]$

no crit. #'s

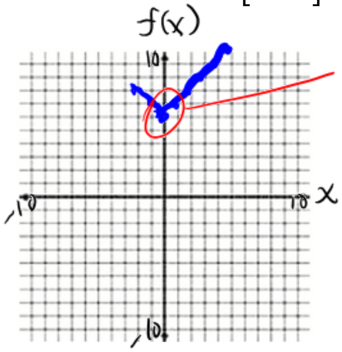
$f(\pi/6) = \frac{\pi}{6}(\tan(\pi/6)) = \frac{\pi}{6}\left(\frac{\sqrt{3}}{3}\right) = \frac{\pi\sqrt{3}}{18}$

$f(\pi/3) = \frac{\pi}{3}(\tan(\pi/3)) = \frac{\pi}{3}(\sqrt{3}) = \frac{\pi\sqrt{3}}{3}$

minimum: $(\pi/6, \pi\sqrt{3}/18) \approx (0.52, 0.30)$

maximum: $(\pi/3, \pi\sqrt{3}/3) \approx (1.05, 1.81)$

6. (5 points) Determine whether the Mean Value Theorem can be applied to $f(x)=|x|+6$ on the closed interval $[-2,5]$. If so, find all values of c such that $f'(c)=\frac{f(b)-f(a)}{b-a}$.



not differentiable at $x=0$

$\boxed{\text{So the Mean Value Theorem cannot be applied}}$

7. (10 points) Consider the function $f(x)=\frac{x^3-x^2-4x+4}{x^2-1}$.

$$f(x)=\frac{x^2(x-1)-4(x-1)}{(x+1)(x-1)}$$

$$f(x)=\frac{(x^2-4)\cancel{(x-1)}}{(x+1)\cancel{(x-1)}}$$

$$f(x)=\frac{x^2-4}{x+1}$$

a. (2 POINTS) List excluded domain values, and describe what they represent on the graph.

$x \neq \pm 1$

At $x=-1$, there is a vertical asymptote, and at $x=1$, there is a hole in the graph.

b. (8 POINTS) Use **calculus** to find the points of inflection and discuss the concavity of the graph of f. **Exact answers only**.

$$f(x)=\frac{x^2-4}{x+1}$$

$$f'(x)=\frac{2x(x+1)-(x^2-4)(1)}{(x+1)^2}$$

$$f'(x)=\frac{2x^2+2x-x^2+4}{(x+1)^2}$$

$$f'(x)=\frac{x^2+2x+4}{(x+1)^2}$$

$$f''(x)=\frac{(2x+2)(x+1)^{\cancel{2}}-(x^2+2x+4)[2\cancel{(x+1)}(1)]}{(x+1)^{\cancel{4}\,3}}$$

$$f''(x)=\frac{2x^2+4x+2-2x^2-4x-8}{(x+1)^3}$$

$$f''(x)=\frac{-6}{(x+1)^3}$$

no zeros

$(-\infty,-1)$; $(-1,1)$; $(1,\infty)$

+ | − | −

−2 −1 0 1 2

$f''(2)=\frac{-6}{-1}>0$

$f''(0)=-6<0$

$f''(2)=\frac{-6}{27}<0$

f is concave downward on $(-1,1)$ and $(1,\infty)$ and concave upward on $(-\infty,-1)$. There are no points of inflection

8. (4 points) Use **calculus** to find all points where relative extrema occur. **Exact answers only**.

$f(x) = x^5 + 5x^3 - 24$

$f'(x) = 5x^4 + 15x^2$

$0 = 5x^2(x^2 + 3)$

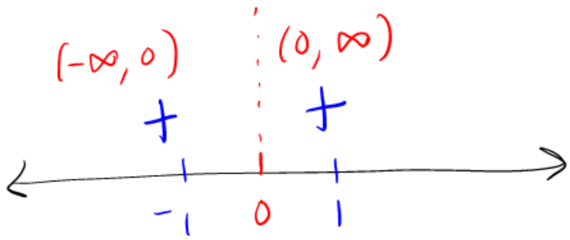
$5x^2 = 0$ or $x^2 + 3 = 0$

$x = 0$ imaginary

$c = 0$

$f''(x) = 20x^3 + 30x$

$f''(0) = 0$ so 2nd deriv. test does not apply



$f'(-1) = 5 + 15 > 0$

$f'(1) = 5 + 15 > 0$

$\boxed{\text{No relative extrema}}$

9. (5 POINTS) Use **differentials** to approximate the value of the expression $\sqrt{35.5}$.

$f(x + \Delta x) \approx f(x) + f'(x)\,dx$

$\Delta x = dx = -0.5$

$x = 36$

$f(x) = \sqrt{x}$

$f'(x) = \dfrac{1}{2\sqrt{x}}$

$f(36 + (-0.5)) \approx f(36) + f'(36)(-0.5)$

$= \sqrt{36} - \dfrac{0.5}{2\sqrt{36}}$

$= 6 - \dfrac{0.5}{2 \cdot 6}$

$= 6 - \dfrac{1}{24}$

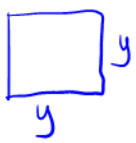
$= \dfrac{144}{24} - \dfrac{1}{24}$

$= \boxed{\dfrac{143}{24}}$

10. (10 POINTS) The sum of the perimeters of an equilateral triangle and a square is 16. Find the dimensions of the triangle and the square that produce a minimum total area (The shapes are not attached). Let *x* represent the measure of the length of each side of the triangle and let *y* represent the measure of each edge of the square. You must use **calculus!**

$2 \cdot 4 \cdot \frac{3}{4}$

a. Analysis (1 POINTS)



$$A_{\triangle} = \frac{1}{2}(x)\left(\frac{\sqrt{3}}{2}x\right) = \frac{\sqrt{3}}{4}x^2$$

$$A_{\square} = y^2$$

$$16 = 3x + 4y \rightarrow y = 4 - \frac{3}{4}x$$

c. Write primary equation as a function of *x* (1 POINT)

$$A(x) = \frac{\sqrt{3}}{4}x^2 + \left(4 - \frac{3}{4}x\right)^2$$

$$\boxed{A(x) = \frac{\sqrt{3}}{4}x^2 + \frac{9}{16}x^2 - 6x + 16}$$

b. Primary Equation (1 POINTS)

$$\boxed{A(x,y) = \frac{\sqrt{3}}{4}x^2 + y^2}$$

d. Feasible Domain

$x > 0,\ y > 0$

$4 - \frac{3}{4}x > 0$

$-\frac{3}{4}x > -4$

$x < \frac{16}{3}$

$$\boxed{0 < x < \frac{16}{3}}$$

e. Minimize

i. Find critical number(s). You must use calculus to find the derivative. You may use your graphing calculator to approximate the zero(s) to the nearest tenth, if necessary. (3 POINTS)

$$A'(x) = \frac{\sqrt{3}}{2}x + \frac{9}{8}x - 6$$

$$0 = \left(\frac{\sqrt{3}}{2} + \frac{9}{8}\right)x - 6$$

$$6 \approx 1.99x$$

$\rightarrow x \approx 3.0$

$c \approx 3.0$

ii. Use the 1st or 2nd derivative test to verify that the critical number produces a relative minimum. Show all steps as we did in class! (2 POINTS)

$$A''(x) = \frac{\sqrt{3}}{2} + \frac{9}{8} > 0$$

So $A''(3) > 0$ which implies that at $x \approx 3$ there's a relative minimum.

iii. Find *y*. (1 POINT)

$$y = 4 - \frac{3}{4}x \rightarrow y \approx 4 - \frac{3}{4}(3.0) \rightarrow \boxed{y \approx 1.8}$$

f. Conclusion in words. (1 POINT)

An equilateral triangle whose sides meaoure approximately 3.0 units and a square whose side meaoure approximately 1.8 units produce the minimum area.