MATH 150/GRACEY NAME_____

EXAM 3/PART 1/50 POINTS POSSIBLE Part 1_____Part 2_____ E3_____%

SCIENTIFIC CALCULATOR ONLY HW_____%Q_____%E_____%

Completely analyze the following function. Be sure to write "NONE" if this function does not have a characteristic listed below.

Consider the function $f(x)=\dfrac{x^2-4}{x^2-x-6}$ $f(x)=\dfrac{(x+2)(x-2)}{(x-3)(x+2)} \rightarrow f(x)=\dfrac{x-2}{x-3}, x\neq -2$

1. Give the ordered pairs representing the intercepts.

$0=x-2$, $x=2$ | $f(0)=\dfrac{0-2}{0-3}=\dfrac{2}{3}$

a. (1 point) *x*-intercept: $(2,0)$

b. (1 point) *y*-intercept: $\left(0,\frac{2}{3}\right)$

2. Write the lines representing the vertical and horizontal asymptotes.

a. (2 points) Vertical asymptote(s): $x=3$

$x-3=0$, $x=3$ $\lim\limits_{x\to\infty}\dfrac{\frac{x-2}{x}}{\frac{x-3}{x}}=\lim\limits_{x\to\infty}\dfrac{1-\frac{2}{x}}{1-\frac{3}{x}}=1=\lim\limits_{x\to-\infty}\dfrac{x-2}{x-3}$

b. (2 points) Horizontal asymptote(s): $y=1$

3. (8 points) Find the critical numbers for f.

$f'(x)=\dfrac{(1)(x-3)-(x-2)(1)}{(x-3)^2}$

$f'(x)=\dfrac{x-3-x+2}{(x-3)^2}$

$f'(x)=\dfrac{-1}{(x-3)^2}$

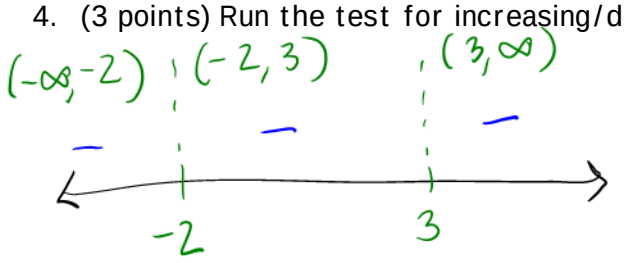
no critical numbers

V.A. at $x=3$

and hole in graph at $x=-2$

$f(x)=\frac{x^2-4}{x^2-x-6}$ $= \frac{x-2}{x-3}$

4. (3 points) Run the test for increasing/decreasing intervals.



$f'(x)=\frac{-1}{(x-3)^2}$ always negative

a. (2 points) f is increasing on NONE

b. (2 points) f is decreasing on $(-\infty,-2)\cup(-2,3)\cup(3,\infty)$

c. (2 points) Give the ordered pairs where relative minima occur.

NONE

d. (2 points) Give the ordered pairs where relative maxima occur.

NONE

5. Test for concavity and find any points of inflection.

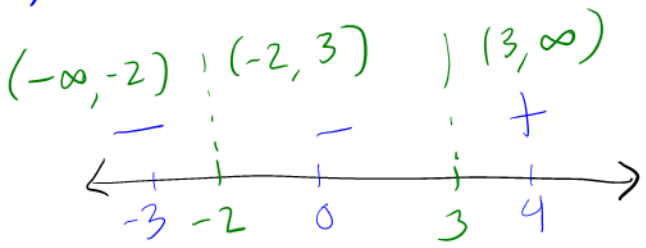
a. (15 points) Run the test for concavity.

$f'(x)=\frac{-1}{(x-3)^2}=-(x-3)^{-2}$

$f''(x)=2(x-3)^{-3}(1)$

$f''(x)=\frac{2}{(x-3)^3}$

no zeros



$f''(-3)=\frac{2}{(-6)^3}<0$

$f''(0)=\frac{2}{(-3)^3}<0$

$f''(4)=\frac{2}{(1)^3}>0$

$$f(x)=\frac{x^2-4}{x^2-x-6}$$

b. (2 points) f is concave upwards on $(3,\infty)$

c. (2 points) f is concave downwards on $(-\infty,-2)\cup(-2,3)$

d. (2 points) Give the ordered pairs which represent points of inflection.

NONE

6. (4 points) Sketch the graph, using the information from your analysis. Find additional ordered pairs as needed. Be sure to label your axes and write in the scale.

$f(x)=\frac{x-2}{x-3}$, $x\neq -2$

Find location of hole:

$f(-2)=\frac{-4}{-5}=\frac{4}{5}$

$f(4)=\frac{4-2}{4-3}=2$

additional points

$f(5)=\frac{3}{2}$

$f(3.5)=\frac{1.5}{.5}=\frac{15}{5}=3$