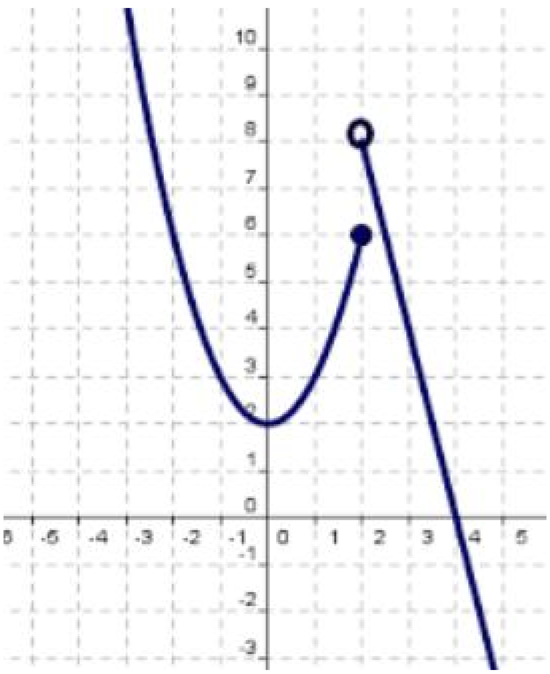
MATH 150/GRACEY
EXAM 1/CHAPTER 1
100 POINTS POSSIBLE

NAME Key

YOUR WORK MUST SUPPORT YOUR ANSWER FOR FULL CREDIT TO BE AWARDED!
NO GRAPHING CALCULATOR AND NO DECIMALS

1. (8 POINTS, 2 POINTS EACH) Use the graph of $y = f(x)$ shown below to find each limit, if it exists. **If the limit does not exist, explain why**.



a. $\lim_{x\to 0} f(x) =$ 2

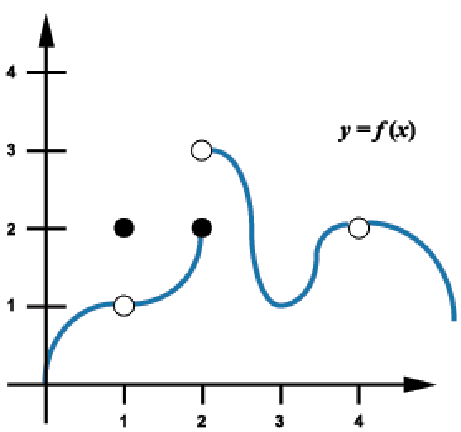
b. $\lim_{x\to 2^-} f(x) =$ 6

c. $\lim_{x\to 2^+} f(x) =$ 8

d. $\lim_{x\to 2} f(x) =$ DNE

$\lim_{x\to 2^-} f(x) \neq \lim_{x\to 2^+} f(x)$

2. (6 POINTS) Consider the function shown below. Is this function continuous at $x = 1$? EXPLAIN using the **3 conditions** for continuity at a point!



1. $f(1) = 2$ ✓
2. $\lim_{x\to 1} f(x) = 1$ ✓
3. $f(1) \neq \lim_{x\to 1} f(x)$ fails

Circle one:

continuous at $x = 1$ not continuous at $x = 1$

3. (6 POINTS) Find the limit L. Then find $\delta > 0$ such that $|f(x) - L| < 0.01$ whenever $0 < |x - c| < \delta$.

$f(x) = 10 - 5x$
$L = -40$
$\varepsilon = 0.01$
$c = 10$

$\lim_{x \to 10} (10 - 5x) = -40$

$|f(x) - L| < \varepsilon$

$|(10 - 5x) - (-40)| < 0.01$

$|50 - 5x| < 0.01$

$|-5(x - 10)| < 0.01$

$\to |-5||x - 10| < 0.01$

$5|x - 10| < 0.01$

$|x - 10| < \frac{0.01}{5}$

$|x - 10| < 0.02$

$\boxed{\delta = 0.0195}$

4. (25 POINTS, 5 POINTS EACH) Find the exact **FINITE LIMIT** analytically. If there is no finite limit, write DNE (does not exist). DO NOT USE YOUR CALCULATOR!

a. $\lim_{x \to \pi} \left(\sec^2(-7x/4)\right)$

D.S. $= \sec^2(-7\pi/4)$

$= \sec^2(\pi/4)$

$= (\sqrt{2})^2 = \boxed{2}$

b. $\lim_{x \to -2} \left(\frac{x^2 - x - 1}{x} \right)$ D.S. $= \frac{(-2)^2 - (-2) - 1}{(-2)}$

$= \frac{4 + 2 - 1}{-2}$

$= \boxed{-5/2}$

c. $\frac{3}{3} \lim_{x \to 0} \frac{1 - \cos 3x}{x} = 3 \lim_{x \to 0} \frac{1 - \cos 3x}{3x}$

$= 3(0)$

$= \boxed{0}$

d. $\lim_{x \to 1/3} \frac{3x - 1}{27x^3 - 1}$

D.S. $\frac{0}{0}$

$= \lim_{x \to 1/3} \frac{\cancel{3x-1}}{(\cancel{3x-1})(9x^2 + 3x + 1)}$

$= \lim_{x \to 1/3} \frac{1}{9x^2 + 3x + 1}$

D.S. $= \frac{1}{9\left(\frac{1}{3}\right)^2 + 3\left(\frac{1}{3}\right) + 1}$

$= \frac{1}{1 + 1 + 1}$

$\to = \boxed{\frac{1}{3}}$

e. $\lim_{x \to -243} \left(-\sqrt[5]{x} + \sqrt[3]{9x} \right)$

D.S. $= -\sqrt[5]{-243} + \sqrt[3]{9(-243)}$

$= -(-3) + \sqrt[3]{3^2(-3)^5}$

$= 3 + \sqrt[3]{(-27)(-27)(-3)}$

$= \boxed{3 - 9\sqrt[3]{3} \approx -9.98}$

5. (16 POINTS, 8 POINTS EACH) Find the exact **FINITE LIMIT** analytically. If there is no finite limit, write DNE (does not exist). DO NOT USE YOUR CALCULATOR!

D.S. $\frac{0}{0}$

a. $\lim_{x \to 1} \frac{\left(\sqrt{x}-1\right)}{\left(x-1\right)} \cdot \frac{\left(\sqrt{x}+1\right)}{\left(\sqrt{x}+1\right)} = \lim_{x \to 1} \frac{\cancel{x-1}}{\cancel{(x-1)}(\sqrt{x}+1)}$

$= \lim_{x \to 1} \frac{1}{\sqrt{x}+1}$

$\overset{D.S.}{=} \frac{1}{\sqrt{1}+1}$

$= \boxed{\frac{1}{2}}$

D.S. $\frac{0}{0}$

b. $\lim_{\Delta x \to 0} \frac{\frac{x}{x} \cdot \frac{4}{\left(x+\Delta x\right)} - \frac{4}{x} \cdot \frac{(x+\Delta x)}{(x+\Delta x)}}{\Delta x} = \lim_{\Delta x \to 0} \frac{4x - 4x - 4\Delta x}{x(x+\Delta x)} \cdot \frac{1}{\Delta x}$

$= \lim_{\Delta x \to 0} \frac{-4\cancel{\Delta x}}{x\cancel{\Delta x}(x+\Delta x)}$

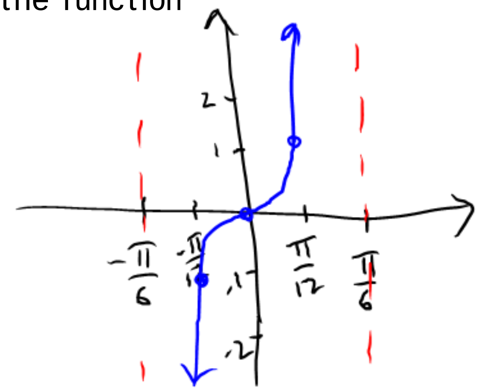
$= \lim_{\Delta x \to 0} \frac{-4}{x(x+\Delta x)}$

$\overset{D.S.}{=} \frac{-4}{x(x+0)}$

$= \boxed{-\frac{4}{x^2}}$

Period: $\frac{\pi}{3}$

6. (7 POINTS) Find the vertical asymptote(s) of the function $f(x) = \tan(3x)$ on $(-\infty, \infty)$.

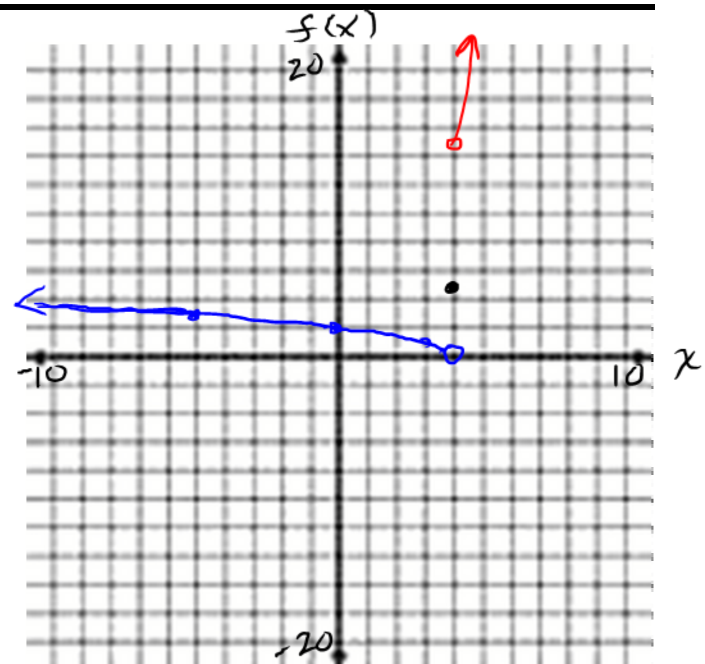


$$\boxed{x = \frac{\pi}{6} + \frac{n\pi}{3},\ n \in \mathbb{Z}}$$

7. (10 POINTS) Consider the function

$$f(x) = \begin{cases} \sqrt{4-x}, & \text{if } x < 4 \\ 5, & \text{if } x = 4 \\ x^2 - 1, & \text{if } x > 4 \end{cases}$$

a) (4 POINTS) Sketch the graph.



b) (3 POINTS) Identify the values of c, for which $\lim_{x \to c} f(x)$ exists. Use interval notation.

$$\boxed{(-\infty, 4) \cup (4, \infty)}$$

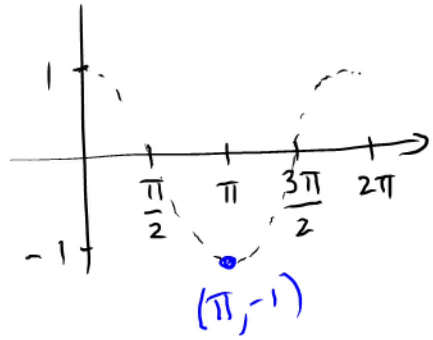
c) (3 POINTS) On what interval(s) is this function continuous? Use interval notation.

$$\boxed{(-\infty, 4) \cup (4, \infty)}$$

8. (10 POINTS, 5 POINTS EACH) Find the limit. It is acceptable to write a result of plus or minus infinity. You may use your graphing calculator.

a. $\lim_{x\to\pi^+} \sec x \overset{D.S.}{=} \sec\pi$

$= \boxed{-1}$



b. $\lim_{x\to -2^+} \dfrac{x-2}{x^2-4} = \boxed{\infty}$

Try -1.99 $\quad \dfrac{-1.99-2}{(-1.99)^2-4} = 100$

---

9. (12 POINTS, 3 POINTS EACH). Evaluate the limits below using the following information:

$$\lim_{x\to c} f(x) = -\infty, \quad \lim_{x\to c} g(x) = \frac{1}{2}, \text{ and } \lim_{x\to c} h(x) = -1$$

a. $\lim_{x\to c}\left[\dfrac{h(x)}{f(x)}\right] = \dfrac{\lim_{x\to c} h(x)}{\lim_{x\to c} f(x)}$

$= \dfrac{-1}{-\infty} = \boxed{0}$

b. $\lim_{x\to c}\left[g(x)f(x)\right]$

$= \left[\lim_{x\to c} g(x)\right]\left[\lim_{x\to c} f(x)\right]$

$= \frac{1}{2}(-\infty) \longrightarrow = \boxed{-\infty}$

c. $\lim_{x\to c}\left(-g(x)+\left[h(x)\right]^2\right)$

$= -\lim_{x\to c} g(x) + \left(\lim_{x\to c} h(x)\right)^2$

$= -\frac{1}{2} + (-1)^2 \longrightarrow = \boxed{\frac{1}{2}}$

d. $\cos^{-1}\left(\lim_{x\to c} g(x)\right) = \cos^{-1}\left(\frac{1}{2}\right)$

$= \boxed{\frac{\pi}{3}}$