MATH 150/GRACEY NAME______________________________________

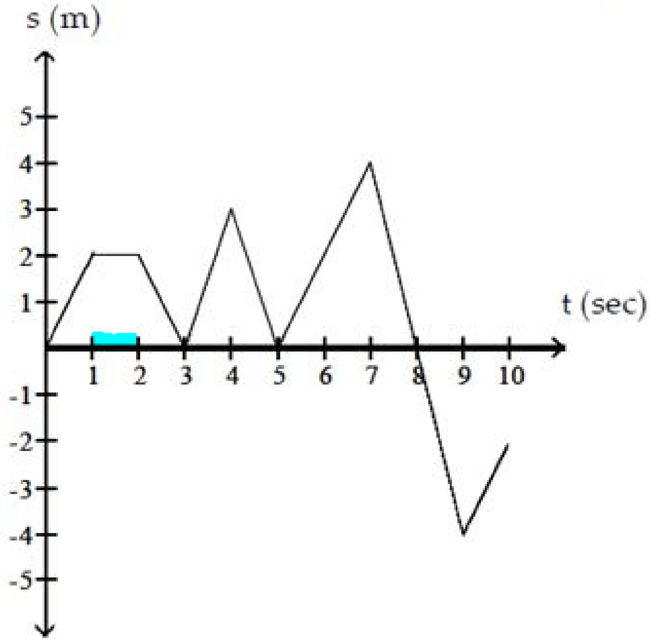
EXAM 2/CHAPTER 2

100 POINTS POSSIBLE/BOX YOUR FINAL ANSWER

SHOW ALL WORK FOR FULL CREDIT!!!

1. (5 POINTS)

The equation gives the position $s = f(t)$ of a body moving on a coordinate line (s in meters, t in seconds).



On (1, 2), the body is at rest.

On what interval(s) is the body at rest?

(48 POINTS) Problems 2-7. Find the derivative of the functions below with respect to the independent variable. Each item is worth 8 points. EXACT, FULLY SIMPLIFIED ANSWERS ONLY!!! This means a single rational expression which has NO COMPLEX FRACTIONS or negative powers.

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2. $f(x) = \dfrac{2x^3 + 4x^2 - x - 2}{x + 2}$

easy way: $f(x) = \dfrac{2x^2(x+2) - 1(x+2)}{x+2}$

$f(x) = \dfrac{\cancel{(x+2)}(2x^2-1)}{\cancel{x+2}}$

$f(x) = 2x^2 - 1$

$\boxed{f'(x) = 4x}$

hardway:

$f'(x) = \dfrac{(6x^2+8x-1)(x+2) - (2x^3+4x^2-x-2)(1)}{(x+2)^2}$

$f'(x) = \dfrac{6x^3+12x^2+8x^2+16x-\cancel{x}-\cancel{2}-2x^3-4x^2+\cancel{x}+\cancel{2}}{(x+2)^2}$

$f'(x) = \dfrac{4x^3+16x^2+16x}{(x+2)^2}$

$f'(x) = \dfrac{4x(x^2+4x+4)}{(x+2)^2}$

$f'(x) = \dfrac{4x\cancel{(x+2)^2}}{\cancel{(x+2)^2}}$

$\boxed{f'(x) = 4x}$

3. $y = \tan^2(3x)$

$$\frac{dy}{dx} = 2(\tan 3x)^1 \cdot \frac{d}{dx}(\tan 3x)$$

$$\frac{dy}{dx} = (2\tan 3x)(\sec^2 3x)\frac{d}{dx}(3x)$$

$$\frac{dy}{dx} = (2\tan 3x)(\sec^2 3x)(3)$$

$$\boxed{\frac{dy}{dx} = 6\tan 3x \sec^2 3x}$$

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4. $h(t) = \dfrac{5-\sqrt{t}}{5+\sqrt{t}}$

$$h'(t) = \frac{(-\frac{1}{2}t^{-1/2})(5+t^{1/2}) - (5-t^{1/2})(\frac{1}{2}t^{-1/2})}{(5+t^{1/2})^2}$$

$$h'(t) = \frac{-\frac{1}{2}t^{-1/2}\left[(5+t^{1/2})+(5-t^{1/2})\right]}{(5+t^{1/2})^2}$$

$$h'(t) = \frac{-10}{2t^{1/2}(5+t^{1/2})^2}$$

$$\boxed{h'(t) = -\frac{5}{t^{1/2}(5+t^{1/2})^2}}$$

5. $f(x) = (x^3 - 1)^{40}$

$f'(x) = 40(x^3-1)^{39}\frac{d}{dx}(x^3-1)$

$f'(x) = 40(x^3-1)^{39}(3x^2)$

$\boxed{f'(x) = 120x^2(x^3-1)^{39}}$

6. $y = x(4-x)^{1/3}$

$\frac{dy}{dx} = 1(4-x)^{1/3} + x\left[\frac{1}{3}(4-x)^{-2/3}\frac{d}{dx}(4-x)\right]$

$\frac{dy}{dx} = (4-x)^{1/3} + x\left[\frac{1}{3}(4-x)^{-2/3}(-1)\right]$

$\frac{dy}{dx} = \frac{1}{3}(4-x)^{-2/3}\left[3(4-x) - x\right]$

$\frac{dy}{dx} = \frac{12-3x-x}{3(4-x)^{2/3}}$

$\frac{dy}{dx} = \frac{12-4x}{3(4-x)^{2/3}}$

$\boxed{\frac{dy}{dx} = \frac{4(3-x)}{3(4-x)^{2/3}}}$

7. $f(\theta) = \frac{\sin 2\theta}{\sin\theta}$

easy way:

$f(\theta) = \frac{2\cancel{\sin\theta}\cos\theta}{\cancel{\sin\theta}}$

$f(\theta) = 2\cos\theta$

$\boxed{f'(\theta) = -2\sin\theta}$

hard way: $f'(\theta) = \frac{(2\cos 2\theta)(\sin\theta) - (\sin 2\theta)(\cos\theta)}{(\sin\theta)^2}$

$f'(\theta) = \frac{2(2\cos^2\theta - 1)\sin\theta - 2\sin\theta\cos\theta\cos\theta}{\sin^2\theta}$

$f'(\theta) = \frac{2\cancel{\sin\theta}(2\cos^2\theta - 1 - \cos^2\theta)}{\sin^{\cancel{2}}\theta}$

$f'(\theta) = \frac{2(\cos^2\theta - 1)}{\sin\theta}$

$f'(\theta) = \frac{-2(1-\cos^2\theta)}{\sin\theta}$

$f'(\theta) = \frac{-2\sin^{\cancel{2}}\theta}{\cancel{\sin\theta}}$

$\boxed{f'(\theta) = -2\sin\theta}$

8. (10 POINTS) Find the derivative with respect to *x*.

$$\frac{d}{dx}\sin(xy) = \frac{d}{dx}\sin(x+y)$$

$$\cos(xy)\frac{d}{dx}(xy) = \cos(x+y)\frac{d}{dx}(x+y)$$

$$(\cos xy)\left(1y + x\frac{dy}{dx}\right) = (\cos(x+y))\left(1+\frac{dy}{dx}\right)$$

$$y\cos xy + x\frac{dy}{dx}\cos xy = \cos(x+y) + \frac{dy}{dx}\cos(x+y)$$

$$x\frac{dy}{dx}\cos xy - \frac{dy}{dx}\cos(x+y) = \cos(x+y) - y\cos xy$$

$$\frac{dy}{dx}(x\cos xy - \cos(x+y)) = \cos(x+y) - y\cos xy$$

$$\boxed{\frac{dy}{dx} = \frac{\cos(x+y) - y\cos xy}{x\cos xy - \cos(x+y)}}$$

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9. (8 POINTS) Find the x-value(s) which yield horizontal tangent lines if $f(x) = 2x^5 - 8x^3 + 2$. **<u>Do not use your graphing calculator</u>.**

$$f'(x) = 10x^4 - 24x^2$$

$$0 = 2x^2(5x^2 - 12)$$

$$\boxed{x = \left\{-\frac{2\sqrt{15}}{5}, 0, \frac{2\sqrt{15}}{5}\right\}}$$

$$2x^2 = 0 \quad \text{or} \quad 5x^2 - 12 = 0$$

$$\sqrt{x^2} = \sqrt{0} \qquad 5x^2 = 12$$

$$x = 0 \qquad \sqrt{x^2} = \sqrt{12/5}$$

$$x = \pm\sqrt{\frac{12}{5}}$$

$$x = \pm\frac{2\sqrt{3}}{\sqrt{5}} \quad \text{or} \quad x = \pm\frac{2\sqrt{15}}{5}$$

10. (8 POINTS) Use the **LIMIT PROCESS** find the derivative of $f(x)=\dfrac{1}{x}$.

D.S. $\frac{0}{0}$

$$f'(x)=\lim_{\Delta x\to 0}\frac{\frac{1}{x+\Delta x}-\frac{1}{x}}{\Delta x}$$

$$f'(x)=\lim_{\Delta x\to 0}\frac{x-(x+\Delta x)}{x(x+\Delta x)}\cdot\frac{1}{\Delta x}$$

$$f'(x)=\lim_{\Delta x\to 0}\frac{\cancel{\Delta x}}{x\cancel{\Delta x}(x+\Delta x)}$$

$$f'(x)=\lim_{\Delta x\to 0}\frac{1}{x(x+\Delta x)}$$

$$f'(x)\overset{D.S.}{=}\frac{1}{x(x+0)}$$

$$\boxed{f'(x)=\frac{1}{x^2}}$$

11. (9 POINTS) A spherical balloon is inflated with helium at the rate of $100\pi\ \text{ft}^3/\text{min}$. How fast is the balloon's radius increasing at the instant the radius is 5 ft? (Given: $V=\frac{4}{3}\pi r^3$)

$$\frac{dV}{dt}=100\pi\ ft^3/min$$

$$r=5$$

$$V=\frac{4}{3}\pi r^3$$

$$\frac{d}{dt}V=\frac{d}{dt}\left(\frac{4}{3}\pi r^3\right)$$

$$\frac{dV}{dt}=\frac{4\pi}{\cancel{3}}\cancel{3}(r)^2\frac{d}{dt}(r)$$

$$\frac{dV}{dt}=4\pi r^2\frac{dr}{dt}$$

$$\frac{dV}{dt}=4\pi r^2\frac{dr}{dt}$$

$$100\pi=4\pi(5)^2\frac{dr}{dt}$$

$$100\pi=100\pi\frac{dr}{dt}$$

$$1=\frac{dr}{dt}$$

The balloon's radius is increasing at a rate of 1 ft/min

12. (12 POINTS) Find the equation(s) of the tangent line(s) to the graph of the ellipse $x^2 + 2y^2 = 17$ at $x = 3$. Please give the linear equation(s) in the point-slope form of the line.

Slope

$$\frac{d}{dx}\left(x^2 + 2y^2\right) = \frac{d}{dx}(17)$$

$$2x + 2\left[2(y)^1 \frac{d}{dx}(y)\right] = 0$$

$$2x + 4y\frac{dy}{dx} = 0$$

$$4y\frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = -\frac{x}{2y}$$

y-coord

$$(3)^2 + 2y^2 = 17$$

$$9 + 2y^2 = 17$$

$$2y^2 = 8$$

$$\sqrt{y^2} = \sqrt{4}$$

$$y = \pm 2$$

At $(3,-2)$:

$$\frac{dy}{dx} = -\frac{3}{2(-2)} = \frac{3}{4}$$

$$\boxed{y + 2 = \frac{3}{4}(x-3)}$$

At $(3,2)$:

$$\frac{dy}{dx} = -\frac{3}{2(2)} = -\frac{3}{4}$$

$$\boxed{y - 2 = -\frac{3}{4}(x-3)}$$