MATH 150/GRACEY NAME Key

EXAM 4/CHAPTER 4 HW____% EXAM____% QUIZ____%

108 ~~100~~ POINTS POSSIBLE/BOX YOUR FINAL ANSWER

SHOW ALL WORK FOR FULL CREDIT!!!

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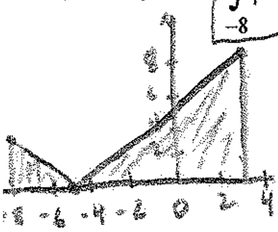
64 (~~56~~ POINTS) Problems 1-8. Evaluate the definite integrals and find the indefinite integrals: Each question is worth 8 points. EXACT ANSWERS ONLY!!!

$A=\frac{1}{2}(3\cdot 3+8\cdot 8)$

$A=\frac{73}{2}$

1. $\int_{-8}^{3}|x+5|\,dx = -\int_{-8}^{-5}(x+5)\,dx + \int_{-5}^{3}(x+5)\,dx$



$= -\left(\frac{x^2}{2}+5x\right]_{-8}^{-5} + \left(\frac{x^2}{2}+5x\right]_{-5}^{3}$

$= -\left[\left(\frac{25}{2}-25\right)-\left(\frac{64}{2}-40\right)\right]+\left[\left(\frac{9}{2}+15\right)-\left(\frac{25}{2}-25\right)\right]$

$= -\left(-\frac{25}{2}-(-8)\right)+\left(\frac{39}{2}-\left(-\frac{25}{2}\right)\right)$

$= -\left(-\frac{9}{2}\right)+\frac{64}{2}$

$= \boxed{\frac{73}{2}}$

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2. $\int 3\theta^5\csc^2\theta^6\,d\theta = \int 3\theta^5\csc^2 u\cdot\frac{du}{6\theta^5}$

$u=\theta^6$

$\frac{du}{d\theta}=6\theta^5$

$d\theta=\frac{du}{6\theta^5}$

$= \frac{1}{2}\int\csc^2 u\,du$

$= \frac{1}{2}(-\cot u)+C$

$= \boxed{-\frac{1}{2}\cot\theta^6+C}$

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3. $\int\frac{x}{\sqrt{1-x}}\,dx = \int\frac{1-u}{u^{1/2}}(-du)$

$u=1-x$

$\frac{du}{dx}=-1$

$dx=-du$

$x=1-u$

$= -\int\left(u^{-1/2}-u^{1/2}\right)du$

$= -\left(2u^{1/2}-\frac{2}{3}u^{3/2}\right)+C$

$= \boxed{-2(1-x)^{1/2}+\frac{2}{3}(1-x)^{3/2}+C}$

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4. $\int\left(1-x^2\right)^3dx = \int\left(1-3x^2+3x^4-x^6\right)dx$

$= \boxed{x-x^3+\frac{3}{5}x^5-\frac{1}{7}x^7+C}$

$\left[1+(-x^2)\right]^3 = (1)^3(-x^2)^0+3(1)^2(-x^2)^1+3(1)^1(-x^2)^2+(1)^0(-x^2)^3$

$= 1-3x^2+3x^4-x^6$

$$\sin^2\frac{x}{2} = \frac{1-\cos 2(x/2)}{2} = \frac{1}{2}(1-\cos x)$$

5. $\int \sin^2\frac{x}{2}\,dx = \frac{1}{2}\int(1-\cos x)\,dx$

$$= \frac{1}{2}(x-\sin x) + C$$

$$= \boxed{\frac{x}{2} - \frac{\sin x}{2} + C}$$

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6. $\int\left(\frac{4x+x^{1/3}}{x^{2/3}}\right)dx = \int\left(4x^{1/3} + x^{-1/3}\right)dx$

$$= 4\cdot\frac{3}{4}x^{4/3} + \frac{3}{2}x^{2/3} + C$$

$$= \boxed{3x^{4/3} + \frac{3}{2}x^{2/3} + C}$$

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7. $\int_3^5 \frac{x^3-1}{x-1}\,dx = \int_3^5 \frac{\cancel{(x-1)}(x^2+x+1)}{\cancel{x-1}}\,dx$

$$= \int_3^5 (x^2+x+1)\,dx$$

$$= \left(\frac{x^3}{3} + \frac{x^2}{2} + x\right|_3^5$$

$$= \left(\frac{125}{3} + \frac{25}{2} + 5\right) - \left(\frac{27}{3} + \frac{9}{2} + 3\right)$$

$$= \frac{98}{3} + 8 + 2$$

$$= \frac{98}{3} + 10$$

$$= \frac{98+30}{3}$$

$$= \boxed{\frac{128}{3}}$$

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8. $\int_{\pi/4}^{\pi/3} \tan^3 x \sec^2 x\,dx = \int_1^{\sqrt{3}} u^3 \cancel{\sec^2 x}\,\frac{du}{\cancel{\sec^2 x}}$

$u = \tan x$

$\frac{du}{dx} = \sec^2 x$

$dx = \frac{du}{\sec^2 x}$

upper lim: $\tan \pi/3 = \sqrt{3}$

lower lim: $\tan \pi/4 = 1$

$$= \int_1^{\sqrt{3}} u^3\,du$$

$$= \frac{u^4}{4}\Big|_1^{\sqrt{3}}$$

$$= \frac{(3^{1/2})^4}{4} - \frac{(1)^4}{4}$$

$$= \frac{9}{4} - \frac{1}{4}$$

$$= \boxed{2}$$

9. (6 POINTS) Find the average value of the function $f(x) = \frac{4}{x^2}$ on the interval $[1,4]$.

Average value $= \frac{1}{b-a}\int_a^b f(x)\,dx$

$= \frac{1}{4-1}\int_1^4 4x^{-2}\,dx$

$= \frac{4}{3}\left(\frac{x^{-1}}{-1}\Big|_1^4\right.$

$= -\frac{4}{3}\left(\frac{1}{4}-\frac{1}{1}\right)$

$= -\frac{4}{3}\left(-\frac{3}{4}\right)$

$= \boxed{1}$

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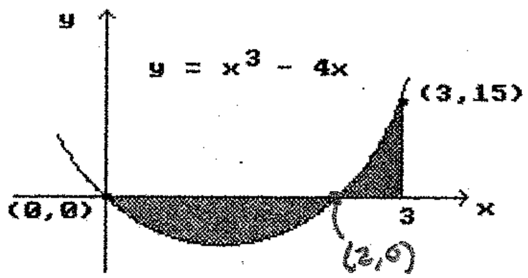
10. (8 POINTS) Find the area of the shaded region.



$x^3-4x=0$

$x(x^2-4)=0$

$x=0,\ x=\pm 2$

$A = -\int_0^2 (x^3-4x)\,dx + \int_2^3 (x^3-4x)\,dx$

$A = -\left(x^4/4 - 2x^2\right|_0^2 + \left(x^4/4 - 2x^2\right|_2^3$

$A = -\left[(4-8)-(0-0)\right] + \left[\left(\frac{81}{4}-18\right)-(4-8)\right]$

$A = -(-4) + \frac{81}{4} - 14$

$A = \frac{81}{4} - 10$

$A = \frac{81-40}{4}$

$\boxed{A = \frac{41}{4} \text{ sq. units}}$

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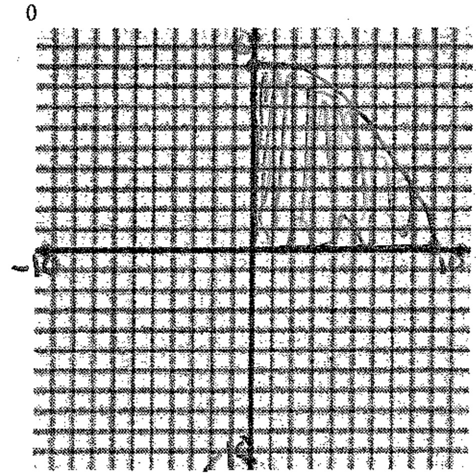
11. (8 POINTS) Sketch the region whose area is given by the definite integral. Then use a geometric formula to evaluate the integral. $\int_0^9 \sqrt{81-x^2}\,dx$.

quarter circle w/radius 9

$A = \frac{1}{4}\pi \cdot 9^2$

$\boxed{A = \frac{81\pi}{4} \text{ sq. units}}$



12. (8 POINTS) Evaluate the definite integral by the **limit definition**.

$$\int_0^3 (1-x^2)\,dx = \lim_{\|\Delta\|\to 0} \sum_{i=1}^{n} f(c_i)\Delta x_i$$

$\Delta x = \frac{b-a}{n} = \frac{3}{n}$

$c_i = a + i\Delta x = \frac{3i}{n}$

$f(c_i) = 1 - \left(\frac{3i}{n}\right)^2 = 1 - \frac{9i^2}{n^2}$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(1 - \frac{9i^2}{n^2}\right)\frac{3}{n}$$

$$= \lim_{n\to\infty} \left[\frac{1}{n}\sum_{i=1}^{n} 3 - \frac{27}{n^3}\sum_{i=1}^{n} i^2\right]$$

$$= \lim_{n\to\infty} \left[\frac{1}{\cancel{n}} 3\cancel{n} - \frac{\overset{9}{\cancel{27}}}{n^{\cancel{3}2}} \cdot \frac{\cancel{n}(n+1)(2n+1)}{\cancel{6}_2}\right] \quad 2n^2+3n+1$$

$$= \lim_{n\to\infty} \left[3 - \frac{18n^2}{2n^2} + \frac{27\cancel{n}}{2n^2} + \frac{9}{2n^2}\right]$$

$$= 3 - 9 + 0 + 0 \quad = \boxed{-6}$$

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13. (8 POINTS) Find the area of the region bounded by $y = \cos x + \sin 2x$, the $x-$axis, $x = 0$ and $x = \frac{\pi}{2}$.

$$A = \int_0^{\pi/2} (\cos x + \sin 2x)\,dx$$

$$A = \left(\sin x - \frac{\cos 2x}{2}\right)\Big|_0^{\pi/2}$$

$$A = \left(\sin \tfrac{\pi}{2} - \frac{\cos \pi}{2}\right) - \left(\sin 0 - \frac{\cos 0}{2}\right)$$

$$A = 1 - \left(\frac{-1}{2}\right) - 0 + \frac{1}{2}$$

$\boxed{A = 2 \text{ sq. units}}$

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14. (6 POINTS) Suppose that g is continuous and that $\int_3^7 g(x)\,dx = 8$ and $\int_3^{12} g(x)\,dx = 5$. Find $\int_{12}^3 g(x)\,dx$.

$$\int_{12}^{3} g(x)\,dx = -\int_{3}^{12} g(x)\,dx$$

$$= \boxed{-5}$$