MATH 150/GRACEY
EXAM 3/PART I/NO CALCULATOR/40 POINTS

EXAM 3

NAME Key

HOMEWORK ______% EXAM______% GRADE______%

Completely analyze the following function. Be sure to write "none" if this function does not have a characteristic listed below.

Consider the function $f(x)=\dfrac{x^2-1}{x^2-9}$.

1. Give the ordered pairs representing the intercepts.
   a. (1 POINT) *x*-intercept: $(-1,0),\ (1,0)$

$0=x^2-1$
$1=x^2$
$\pm 1=x$

   b. (1 POINT) *y*-intercept: $(0, 1/9)$

$f(0)=\dfrac{0^2-1}{0^2-9}$

$f(0)=\dfrac{1}{9}$

2. Write the lines representing the horizontal and vertical asymptotes.

   a. (1 POINT) Vertical asymptote(s): $x=-3,\ x=3$

$x^2-9=0 \Rightarrow x=\pm 3$
$x^2=9$

   b. (3 POINTS) Horizontal asymptote(s): $y=1$

$$\lim_{x\to\infty}\frac{\frac{x^2}{x^2}-\frac{1}{x^2}}{\frac{x^2}{x^2}-\frac{9}{x^2}}=\lim_{x\to\infty}\frac{1-\frac{1}{x^2}}{1-\frac{9}{x^2}}$$

$$=\frac{1-0}{1-0}$$

$$=1$$

3. (5 POINTS) Find the critical number(s) for *f*.

$$f'(x) = \frac{2x(x^2-9) - (x^2-1)(2x)}{(x^2-9)^2}$$

$$f'(x) = \frac{2x[(x^2-9) - (x^2-1)]}{(x^2-9)^2}$$

$$f'(x) = \frac{2x(-8)}{(x^2-9)^2}$$

$$f'(x) = \frac{-16x}{(x^2-9)^2}$$

$0 = -16x$

$$\boxed{\begin{array}{l} 0 = x \\ \text{and} \\ \text{not diff. at } x = \pm 3 \end{array}}$$

4. Run the test for increasing/decreasing intervals of *f* and use the first derivative test to locate any relative extrema.

   a. (3 POINTS) Test for increasing/decreasing functions.

$(-\infty,-3)$ | $(-3,0)$ | $(0,3)$ | $(3,\infty)$

$+$ | $+$ | $-$ | $-$

-3 | 0 | 3

$f'(-4) > 0$

$f'(-1) > 0$

$f'(1) < 0$

$f'(4) < 0$

   b. (2 POINTS) *f* is increasing on $(-\infty,-3) \cup (-3,0)$

   c. (2 POINTS) *f* is decreasing on $(0,3) \cup (3,\infty)$

   d. (2 POINTS)Give the ordered pair(s) where any relative minima occur.

NONE

   e. (2 POINTS)Give the ordered pair(s) where any relative maxima occur.

$$(0, f(0)) = \boxed{\left(0, \tfrac{1}{9}\right)}$$

5. Run the test for concavity and find any points of inflection.
   a. (8 POINTS) Test for concavity.

$$f'(x) = \frac{-16x}{(x^2-9)^2}$$

$$f''(x) = \frac{-16(x^2-9)^{\cancel{2}\,1} + 16x\left[2(\cancel{x^2-9})\cdot 2x\right]}{(x^2-9)^{\cancel{4}\,3}}$$

$$f''(x) = \frac{-16\left[(x^2-9) - 4x^2\right]}{(x^2-9)^3}$$

$$f''(x) = \frac{-16(-3x^2-9)}{(x^2-9)^3}$$

$$f''(x) = \frac{16(3x^2+9)}{(x^2-9)^3}$$

$0 = 16(3x^2+9)$

no real zeros so use $x = \pm 3$

$(-\infty,-3)$ | $(-3,3)$ | $(3,\infty)$
$+$ | $-$ | $+$
-3, 3

$f''(-4) > 0$
$f''(0) < 0$
$f''(4) > 0$

   b. (2 POINTS) f is concave upwards on $(-\infty,-3) \cup (3,\infty)$.

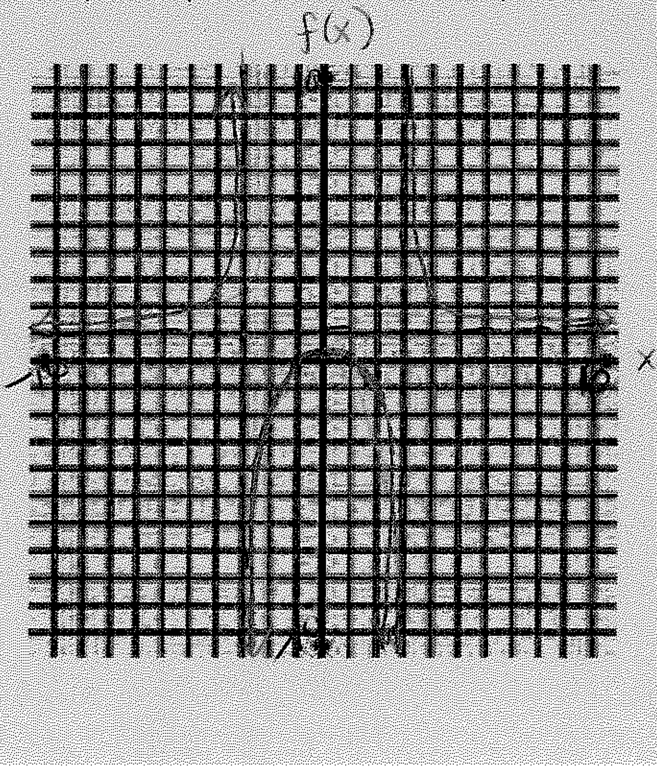
   c. (2 POINTS) f is concave downwards on $(-3,3)$.

   d. (2 POINTS)Give the ordered pairs which represent points of inflection.

$\boxed{\text{NONE}}$

6. (4 POINTS) Sketch the graph, using the information from your analysis. Find additional ordered pairs as needed. Be sure to label your axes and write in the scale.

$f(-4) = \frac{15}{7}$
$f(4) = \frac{15}{7}$



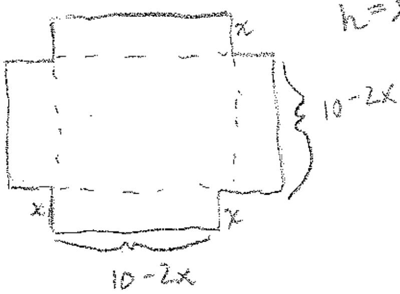
NAME Key

1. (15 POINTS) From a thin piece of cardboard 10 in. by 10 in., square corners are cut out so that the sides can be folded up to make a box. What dimensions will yield a box of maximum volume? What is the maximum volume? Round to the nearest tenth, if necessary.

Step1: Analysis

$l = w = 10-2x$

$h = x$



Step2: Primary Equation

$V(l, w, h) = l \cdot w \cdot h$

Step3: Reduce Primary

$V(x) = (10-2x)(10-2x)(x)$

$V(x) = x(10-2x)^2$

Step4: Domain

$x > 0$ and $10-2x > 0$

$0 < x < 5$

Step5: Optimize

$V'(x) = 1(10-2x)^2 + x(2(10-2x)(-2))$

$V'(x) = (10-2x)[(10-2x)-4x]$

$V'(x) = (10-2x)(10-6x)$

$V'(x) = 4(5-x)(5-3x)$

$0 = 4(5-x)(5-3x)$

$x = 5$ or $x = \frac{5}{3}$

$+ \quad -$

$0 \quad \frac{5}{3} \quad 5$

$V'(1) > 0$

$V'(4) < 0$

so rel. max at $x = \frac{5}{3}$

Step6: Conclusion

$l = 10-2x$

$l = 10-2(\frac{5}{3})$

$l = \frac{20}{3} = w$

The dimensions are $(\frac{20}{3} \times \frac{20}{3} \times \frac{5}{3})$ in and the max. volume is $\frac{2000}{27}$ in^3 OR approx $(6.7 \times 6.7 \times 1.7)$ and 74.1 in^3

2. (10 POINTS) Find the absolute extreme values of the function and the **ordered pairs** where they occur. $f(x) = 3\sqrt{x} - x$ on the closed interval $[0, 9]$. Exact answers only.

$f'(x) = \frac{3}{2\sqrt{x}} - 1$

$0 = \frac{3}{2\sqrt{x}} - 1$

$1 = \frac{3}{2\sqrt{x}}$

$2\sqrt{x} = 3$

$(\sqrt{x})^2 = (3/2)^2$

$x = \frac{9}{4}$

$f(0) = 3\sqrt{0} - 0$

$f(0) = 0$

$f(\frac{9}{4}) = 3\sqrt{\frac{9}{4}} - \frac{9}{4}$

$f(\frac{9}{4}) = 3 \cdot \frac{3}{2} - \frac{9}{4}$

$f(\frac{9}{4}) = \frac{18}{4} - \frac{9}{4}$

$f(\frac{9}{4}) = \frac{9}{4}$

$f(9) = 3\sqrt{9} - 9$

$f(9) = 9 - 9$

$f(9) = 0$

a. Absolute minimum: $(0,0)$ and $(9,0)$

b. Absolute maximum: $(\frac{9}{4}, \frac{9}{4})$

3. (5 POINTS)Use **differentials** to approximate the value of the expression $\sqrt[3]{26}$.

$f(x+\Delta x) = f(x) + \Delta y$

$f(x+\Delta x) \approx f(x) + f'(x)dx$

$f(27+(-1)) \approx \sqrt[3]{27} + \frac{1}{3}(27)^{-2/3}(-1)$

$\to f(27+(-1)) \approx 3 - \frac{1}{27}$

$\boxed{f(27+(-1)) \approx \frac{80}{27}}$

$f(x) = \sqrt[3]{x}$

$f'(x) = \frac{1}{3}x^{-2/3}$

$x = 27$

$\Delta x = dx = -1$

4. (10 POINTS) Sketch the graph of a function f having the following characteristics:

$f(0) = f(6) = 0$

$f'(3) = f'(5) = 0$

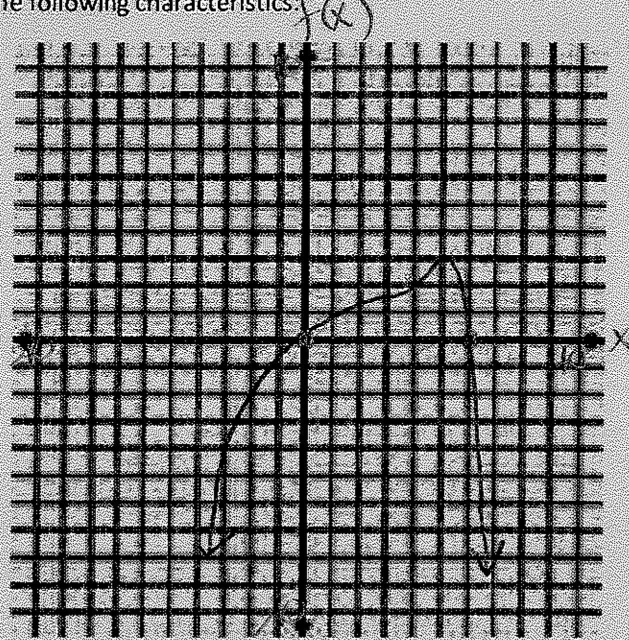
$f'(x) > 0$ if $x < 3$

$f'(x) > 0$ if $3 < x < 5$

$f'(x) < 0$ if $x > 5$

$f''(x) < 0$ if $x < 3$ or $x > 4$

$f''(x) > 0$ if $3 < x < 4$



5. (10 POINTS)The radius of a spherical balloon is measured as 8 inches, with a possible error of 0.02 inch. Use differentials to approximate the maximum possible error in calculating the volume of the sphere. Round to the nearest hundredth, if necessary.

$r = 8$ in

$dr = \pm 0.02$ in

$V = \frac{4}{3}\pi r^3$

$dV = \frac{4}{3}\pi \cdot 3r^2 dr$

$dV = 4\pi(8)^2(\pm 0.02)$

$dV = \pm 16.08$ in^3

The maximum possible error in calculating the volume of the sphere is approximately ± 16.08 in^3.

6. (10 POINTS) Determine whether Rolle's Theorem can be applied to $f(x) = 2x^2 - 7$, $[0,4]$ If Rolle's Theorem can be applied, find all values of c on $(0,4)$ such that $f'(c) = 0$. If Rolle's Theorem cannot be applied, explain why not.

$f(0) = -7$

$f(4) = 25$

Since $f(0) \neq f(4)$
So Rolle's Theorem cannot be applied

extra credit