MATH 150/GRACEY NAME Key

EXAM 2/CHAPTER 2

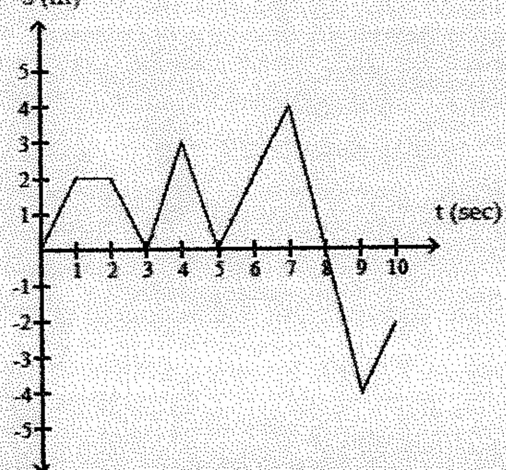
100 POINTS POSSIBLE/BOX YOUR FINAL ANSWER

SHOW ALL WORK FOR FULL CREDIT!!!

1. (5 POINTS)

The equation gives the position s = f(t) of a body moving on a coordinate line (s in meters, t in seconds)

5)



When is the body moving forward?

$$\boxed{(0,1)\cup(3,4)\cup(5,7)\cup(9,10)}$$

(48 POINTS) Problems 2-7. Find the derivative of the functions below with respect to the independent variable. Each item is worth 8 points. EXACT, FULLY SIMPLIFIED ANSWERS ONLY!!! This means a single rational expression which has NO COMPLEX FRACTIONS or negative powers.

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2. $f(x)=\dfrac{x^2+2x+3\sqrt{x}}{\sqrt[3]{x^2}}$

$f(x)=x^{4/3}+2x^{1/3}+3x^{-1/6}$

$f'(x)=\frac{4}{3}x^{1/3}+\frac{2}{3}x^{-2/3}-\frac{3}{6}x^{-7/6}$

$f'(x)=\frac{x^{-7/6}}{6}\left(8x^{3/2}+4x^{1/2}-3\right)$

$$\boxed{f'(x)=\frac{8x^{3/2}+4x^{1/2}-3}{6x^{7/6}}}$$

3. $y = \cos^2\left(\sqrt{x}\right)$

$y = [\cos(x^{1/2})]^2$

$y' = 2[\cos(x^{1/2})]^1 (-\sin(x^{1/2}))\ \frac{1}{2\sqrt{x}}$

$$\boxed{y' = -\frac{\cos\sqrt{x}\sin\sqrt{x}}{\sqrt{x}} \quad \text{or} \quad y' = -\frac{\sin 2\sqrt{x}}{2\sqrt{x}}}$$

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4. $h(t) = \dfrac{5-\sqrt{t}}{5+\sqrt{t}}$

$$h'(t) = \frac{-\frac{1}{2}t^{-1/2}(5+t^{1/2}) - (5-t^{1/2})\frac{1}{2}t^{-1/2}}{(5+t^{1/2})^2}$$

$$h'(t) = \frac{-\frac{1}{2}t^{-1/2}\left[(5+t^{1/2}) + (5-t^{1/2})\right]}{(5+t^{1/2})^2}$$

$$h'(t) = -\frac{5+t^{1/2}+5-t^{1/2}}{2t^{1/2}(5+t^{1/2})^2}$$

$$h'(t) = -\frac{10}{2t^{1/2}(5+t^{1/2})^2}$$

$$\boxed{h'(t) = -\frac{5}{t^{1/2}(5+t^{1/2})^2}}$$

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5. $f(x) = \sqrt{\sqrt{x}+7}$

$f(x) = (x^{1/2}+7)^{1/2}$

$f'(x) = \frac{1}{2}(x^{1/2}+7)^{-1/2} \cdot \frac{1}{2}x^{-1/2}$

$$\boxed{f'(x) = \frac{1}{4x^{1/2}(x^{1/2}+7)^{1/2}}}$$

6. $y=x(4-x)^{1/3}$

$$\frac{dy}{dx}=1(4-x)^{1/3}+x\left[\tfrac{1}{3}(4-x)^{-2/3}(-1)\right]$$

$$\frac{dy}{dx}=\frac{(4-x)^{-2/3}}{3}\left[3(4-x)^{1}-x\right]$$

$$\frac{dy}{dx}=\frac{12-3x-x}{3(4-x)^{2/3}}$$

$$\frac{dy}{dx}=\frac{12-4x}{3(4-x)^{2/3}}$$

$$\boxed{\frac{dy}{dx}=\frac{4(3-x)}{3(4-x)^{2/3}}}$$

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7. $f(\theta)=\dfrac{\csc\theta}{\theta}$

$$f'(\theta)=\frac{(-\csc\theta\cot\theta)\,\theta-\csc\theta\,(1)}{\theta^2}$$

$$\boxed{f'(\theta)=-\frac{\csc\theta(\theta\cot\theta+1)}{\theta^2}}$$

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8. (8 POINTS) Find the derivative with respect to *x*.

$\frac{d}{dx}\tan(x+y)=\frac{d}{dx}x$

$$\sec^2(x+y)\left(1+\frac{dy}{dx}\right)=1$$

$$1+\frac{dy}{dx}=\cos^2(x+y)$$

$$\boxed{\frac{dy}{dx}=-1+\cos^2(x+y)}$$

OR

$$\boxed{\frac{dy}{dx}=-\sin^2(x+y)}$$

DIDN'T NEED!

$$\cancel{\frac{d^2y}{dx^2}=0+2\cos(x+y)\left(1+\frac{dy}{dx}\right)}$$

$$\cancel{\frac{d^2y}{dx^2}=2\cos(x+y)\left[1+(-1+\cos^2(x+y))\right]}$$

$$\cancel{\frac{d^2y}{dx^2}=2\cos(x+y)(\cos^2(x+y))}$$

$$\cancel{\boxed{\frac{d^2y}{dx^2}=2\cos^3(x+y)}}$$

9. (10 POINTS) Find the x-values which yield horizontal tangent lines over the interval $(0,\pi)$ if $f(x)=2\cos x+\sin 2x$.

$f'(x) = -2\sin x + (\cos 2x)(2)$

$0 = -2(\sin x - \cos 2x)$

$0 = \sin x - \cos 2x$

$0 = \sin x - (1 - 2\sin^2 x)$

$0 = 2\sin^2 x + \sin x - 1$

$0 = (2\sin x - 1)(\sin x + 1)$

$2\sin x - 1 = 0$ or $\sin x + 1 = 0$

$\sin x = \frac{1}{2}$ or $\sin x = -1$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$ $\quad$ $\cancel{x = \frac{3\pi}{2}}$

$\boxed{\left\{\frac{\pi}{6}, \frac{5\pi}{6}\right\}}$

10. (10 POINTS) Find $\frac{d^2y}{dx^2}$ of $1-xy=x-y$ in terms of x and y.

$\frac{d}{dx}(1 - xy) = \frac{d}{dx}(x - y)$

$-\left(1y + x\frac{dy}{dx}\right) = 1 - \frac{dy}{dx}$

$-y - x\frac{dy}{dx} = 1 - \frac{dy}{dx}$

$\frac{dy}{dx} - x\frac{dy}{dx} = 1 + y$

$\frac{dy}{dx}(1-x) = 1+y$

$\frac{dy}{dx} = \frac{1+y}{1-x}$

$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(\frac{1+y}{1-x}\right)$

$\frac{d^2y}{dx^2} = \frac{\frac{dy}{dx}(1-x) - (1+y)(-1)}{(1-x)^2}$

$\frac{d^2y}{dx^2} = \frac{\frac{1+y}{\cancel{1-x}}\cancel{(1-x)} + (1+y)}{(1-x)^2}$

$\boxed{\frac{d^2y}{dx^2} = \frac{2(1+y)}{(1-x)^2}}$

11. (8 POINTS) The curve $y = ax^2 + bx + c$ passes through the point $(2,8)$ and is tangent to the line $y = 2x$ at the origin. Find *a, b,* and *c.*

At $(2,8)$: $8 = a(2)^2 + b(2) + c$ At $(0,0)$: $0 = a\cdot 0^2 + b\cdot 0 + c$

$8 = 4a + 2b + c$ $0 = c$

$y' = 2ax + b$ and the slope of the tangent line at $(0,0)$ is 2 $\left[\begin{matrix} y=2x, \\ m=2 \end{matrix}\right]$

So at $(0,0)$ $2 = 2a\cdot 0 + b$

$2 = b$

thus $8 = 4a + 2b + c$ gives us $8 = 4a + 2(2) + 0$

$8 = 4a + 4$

$4 = 4a$

$1 = a$

Hence, $\boxed{y = x^2 + 2x}$

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12. (5 POINTS) A spherical balloon is inflated with helium at the rate of $100\pi \text{ ft}^3/\text{min}$. How fast is the balloon's radius increasing at the instant the radius is 5 ft? (Given: $V = \frac{4}{3}\pi r^3$)

$\frac{dV}{dt} = 100\pi \text{ ft}^3/\text{min}$

we want $\frac{dr}{dt}$ exactly when $r = 5$ ft

$\frac{d}{dt}(V) = \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right)$

$\frac{dV}{dt} = \frac{4}{\cancel{3}}\pi \cdot \cancel{3}r^2 \cdot \frac{dr}{dt}$

$\frac{100\pi \text{ ft}^3}{\text{min}} = 4\pi \cdot (5 \text{ ft})^2 \frac{dr}{dt}$

$\frac{100\pi \text{ ft}^3}{\text{min}} = 100\pi \text{ ft}^2 \frac{dr}{dt}$

$\frac{\cancel{100\pi} \text{ ft}^{\cancel{3}}}{\text{min}} \cdot \frac{1}{\cancel{100\pi \text{ ft}^2}} = \frac{dr}{dt}$

$\boxed{\frac{dr}{dt} = 1 \text{ ft}/\text{min}}$

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13. (6 POINTS) Use the **LIMIT PROCESS** find the derivative of $f(x) = \sin x$.

$f'(x) = \lim_{\Delta x \to 0} \frac{\sin(x + \Delta x) - \sin x}{\Delta x}$

$f'(x) = \lim_{\Delta x \to 0} \frac{\sin x \cos\Delta x + \cos x \sin\Delta x - \sin x}{\Delta x}$

$f'(x) = \lim_{\Delta x \to 0} \frac{-\sin x(1 - \cos\Delta x)}{\Delta x} + \lim_{\Delta x \to 0} \frac{\sin\Delta x \cos x}{\Delta x}$

$f'(x) = -\sin x(0) + (1)\cos x$

$\boxed{f'(x) = \cos x}$