MATH 150/GRACEY NAME Key

EXAM 1/CHAPTER 1

~~100~~ POINTS POSSIBLE

97

YOUR WORK MUST SUPPORT YOUR ANSWER FOR FULL CREDIT TO BE AWARDED!

NO CALCULATORS!!!

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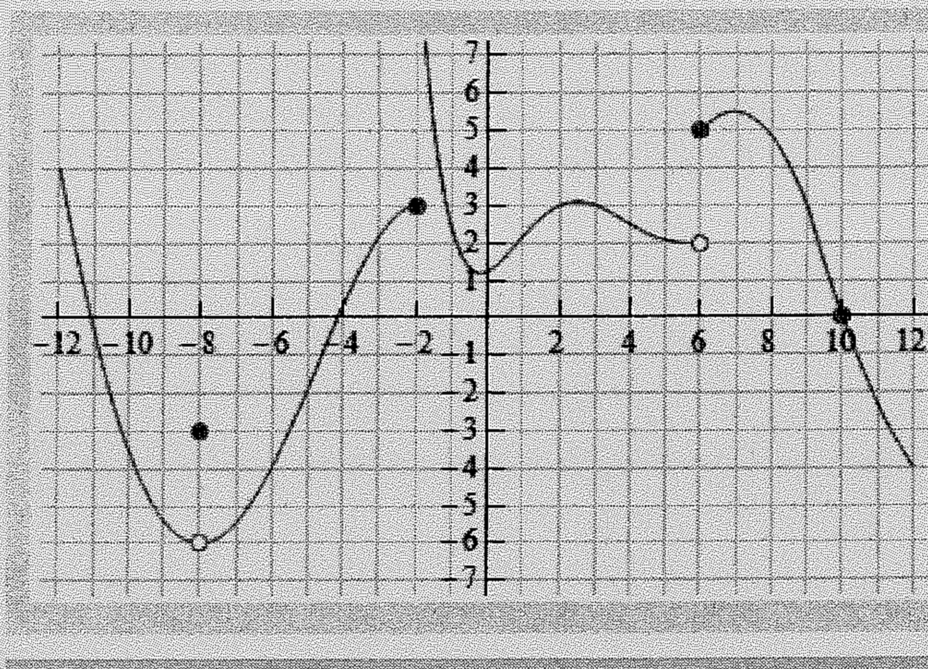
1. (8 POINTS, 2 POINTS EACH) Use the graph of $y = f(x)$ shown below to find each limit, if it exists. If the limit does not exist, explain why.



a. $\lim_{x \to -8} f(x) = \boxed{-6}$

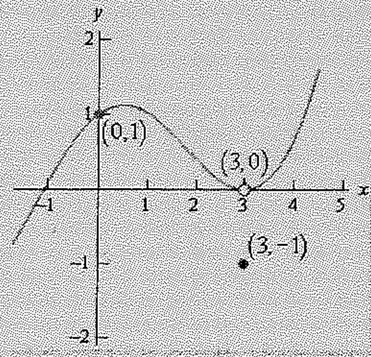
c. $\lim_{x \to 6^+} f(x) = \boxed{5}$

b. $\lim_{x \to 6^-} f(x) = \boxed{2}$

d. $\lim_{x \to 6} f(x)$

DNE $\rightarrow \lim_{x \to 6^-} f(x) \neq \lim_{x \to 6^+} f(x)$

2. (6 POINTS) Consider the function shown below. Is this function continuous at $x = 3$? EXPLAIN using the **3 conditions** for continuity at a point!



① $f(3) = -1$ ✓

② $\lim\limits_{x \to 3} y = 0$ ✓

③ $f(3) \neq \lim\limits_{x \to 3} y$ FAILS

$\boxed{\text{No} \to \text{fails condition 3}}$

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3. (6 POINTS) Find the limit L. Then find $\partial > 0$ such that $|f(x) - L| < 0.01$ whenever $0 < |x - c| < \partial$.

$$\lim_{x \to 2}\left(4 - \frac{x}{2}\right) = 3$$

$f(x) = 4 - \frac{x}{2}$

$L = 3$

$c = 2$

$\varepsilon = 0.01$

$|f(x) - L| < \varepsilon$

$|4 - \frac{x}{2} - 3| < 0.01$

$|1 - \frac{x}{2}| < 0.01$

$\left|\frac{2-x}{2}\right| < 0.01$

$|-\frac{1}{2}(x-2)| < 0.01$

$|-\frac{1}{2}||x-2| < 0.01$

$|x-2| < 0.02 = \delta$

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4. (25 POINTS, 5 POINTS EACH) Find the exact **FINITE LIMIT** analytically. If there is no finite limit, write DNE (does not exist). DO NOT USE YOUR CALCULATOR!

a. $\lim\limits_{x \to \pi}\left(\sin \frac{7x}{3}\right) = \sin \frac{7\pi}{3}$

$= \sin \frac{\pi}{3}$

$= \boxed{\sqrt{3}/2}$

b. $\lim\limits_{x \to 7\pi/4} \csc x = \csc \frac{7\pi}{4}$

$= \boxed{-\sqrt{2}}$

c. $\lim\limits_{x \to 0} \frac{\tan^2 x}{x} = \lim\limits_{x \to 0} \frac{\frac{\sin^2 x}{\cos^2 x}}{x}$

D.S. $\frac{0}{0}$

$= \lim\limits_{x \to 0} \frac{\sin x}{x} \lim\limits_{x \to 0} \frac{\sin x}{\cos^2 x}$

$= (1)\left(\frac{0}{1^2}\right)$

$= \boxed{0}$

d. $\lim\limits_{x \to 1} \frac{x-1}{x^3 - 1} = \lim\limits_{x \to 1} \frac{\cancel{x-1}}{\cancel{(x-1)}(x^2+x+1)}$

D.S. $\frac{0}{0}$

$= \lim\limits_{x \to 1} \frac{1}{x^2+x+1}$

$= \frac{1}{(1)^2 + (1) + 1}$

$= \boxed{\frac{1}{3}}$

e. $\lim\limits_{x \to 400}\left(5x - \sqrt{x}\right) = 5(400) - \sqrt{400}$

$= 2000 - 20$

$= \boxed{1980}$

6

5. (12 POINTS, 7 POINTS EACH) Find the exact **FINITE LIMIT** analytically. If there is no finite limit, write DNE (does not exist). DO NOT USE YOUR CALCULATOR!

D.S. $\frac{0}{0}$

a. $$\lim_{x\to 3}\frac{(\sqrt{x+1}-2)(\sqrt{x+1}+2)}{(x-3)(\sqrt{x+1}+2)} = \lim_{x\to 3}\frac{x+1-4}{(x-3)(\sqrt{x+1}+2)}$$

$$= \lim_{x\to 3}\frac{\cancel{x-3}^{\,1}}{\cancel{(x-3)}_{1}(\sqrt{x+1}+2)}$$

$$= \lim_{x\to 3}\frac{1}{\sqrt{x+1}+2}$$

$$= \frac{1}{\sqrt{3+1}+2}$$

$$= \boxed{\frac{1}{4}}$$

b. $$\lim_{\Delta x\to 0}\frac{(x+\Delta x)^2 - x^2}{\Delta x}$$

D.S. $\frac{0}{0}$

difference of squares

$$= \lim_{\Delta x\to 0}\frac{[(x+\Delta x)+x][(x+\Delta x)-x]}{\Delta x}$$

$$= \lim_{\Delta x\to 0}\frac{(2x+\Delta x)\cancel{\Delta x}^{\,1}}{\cancel{\Delta x}_{1}}$$

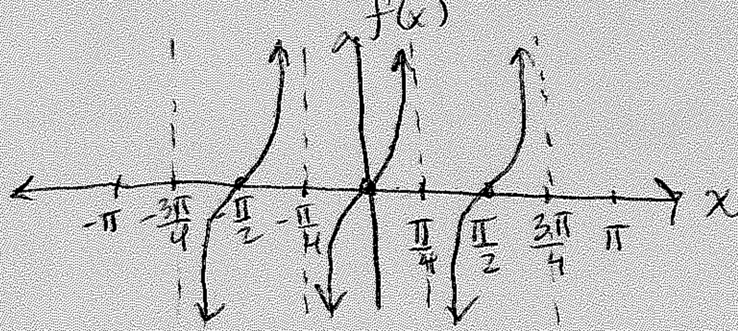
$$= \lim_{\Delta x\to 0}(2x+\Delta x)$$

$$= 2x+0$$

$$= \boxed{2x}$$

6. (8 POINTS) Find the vertical asymptote(s) of the function

$f(x) = \tan 2x$ on $(-\infty, \infty)$.



Vertical Asymptotes are located at $x = \frac{\pi}{4} + \frac{\pi}{2} \cdot k$, $k \in \mathbb{Z}$

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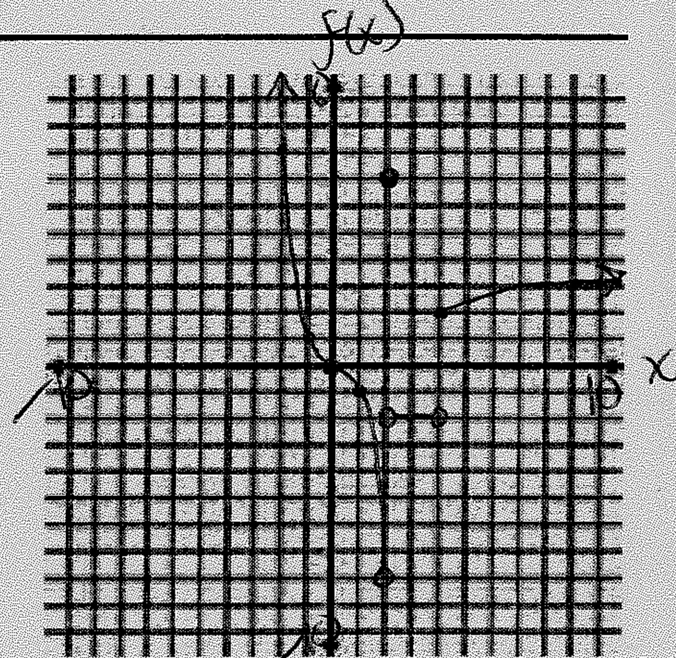
7. (10 POINTS) Consider the function

$$f(x) = \begin{cases} -x^3, & \text{if } x < 2, \\ 7, & \text{if } x = 2, \\ -2, & \text{if } 2 < x < 4 \\ \sqrt{x}, & \text{if } x \geq 4 \end{cases}$$

a) (4 POINTS) Sketch the graph.



b) (3 POINTS) Identify the values of c, for which $\lim_{x \to c} f(x)$ exists. Use interval notation.

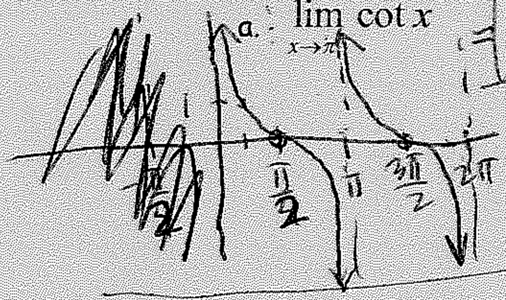
$(-\infty, 2) \cup (2, 4) \cup (4, \infty)$

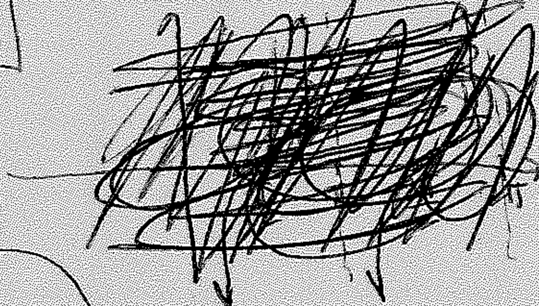
c) (3 POINTS) On what interval(s) is this function continuous? Use interval notation.

$(-\infty, 2) \cup (2, 4) \cup (4, \infty)$

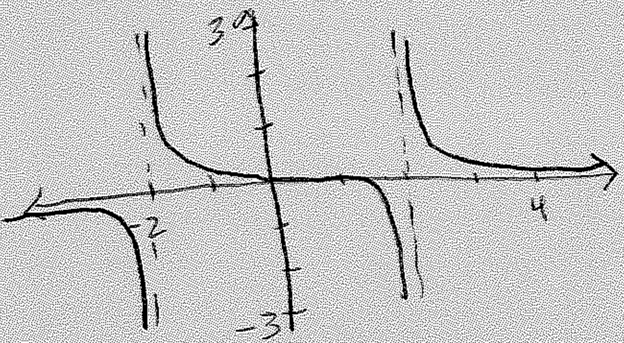
8. (10 POINTS, 5 POINTS EACH) Find the limit. It is acceptable to write a result of plus or minus infinity. Show work by making a table of values or sketching the graph that corresponds to the limit.

a. $\lim\limits_{x \to \pi^-} \cot x = \boxed{+\infty}$





b. $\lim\limits_{x \to 2^+} \dfrac{x}{x^2 - 4} = \boxed{\infty}$



9. (12 POINTS, 3 POINTS EACH). Evaluate the limits below using the following information:

$$\lim_{x \to c} f(x) = \infty,\ \lim_{x \to c} g(x) = -\frac{1}{\sqrt{2}},\ \text{and}\ \lim_{x \to c} h(x) = -3$$

a. $\lim\limits_{x \to c} \left[\dfrac{g(x)}{2f(x)}\right]^2 = \dfrac{\left(-\frac{1}{\sqrt{2}}\right)^2}{2 \cdot \infty}$

$= \boxed{0}$

b. $\lim\limits_{x \to c} \left[g(x) f(x)\right] = -\dfrac{1}{\sqrt{2}} \cdot \infty$

$= \boxed{-\infty}$

c. $\lim\limits_{x \to c} \left(-g(x) + \left[h(x)\right]^2\right)$

$= -\left(-\frac{1}{\sqrt{2}}\right) + (-3)^2$

$= \boxed{\frac{1}{\sqrt{2}} + 9 \text{ or } \frac{1 + 9\sqrt{2}}{\sqrt{2}}}$

d. $\sin^{-1}\left(\lim\limits_{x \to c} g(x)\right)$

$= \sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

$= \boxed{-\frac{\pi}{4}}$