NAME Key

CALCULUS I/MATH 150
SHANNON MYERS

PLEASE MAKE SURE YOU ARE TAKING THE CORRECT EXAM FOR YOUR CLASS!!!

- π 100 POINTS POSSIBLE
- π YOUR WORK MUST SUPPORT YOUR ANSWER FOR FULL CREDIT TO BE AWARDED
- π YOU MAY USE A SCIENTIFIC CALCULATOR ONLY
- π PROVIDE EXACT ANSWERS UNLESS OTHERWISE INDICATED
- π NO NOTES

ONCE YOU BEGIN THE EXAM, YOU MAY NOT LEAVE THE CLASSROOM/PROCTORING CENTER UNTIL YOU ARE FINISHED...THIS MEANS NO BATHROOM BREAKS UNLESS IT IS AN EMERGENCY!

MATH 150/MYERS NAME Key

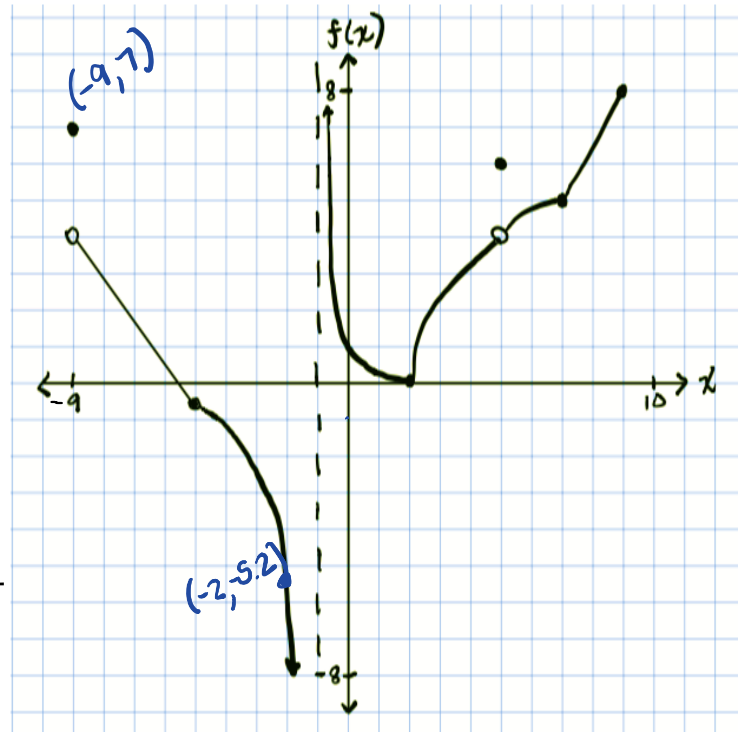
EXAM 1/CHAPTER 1-2.1

100 POINTS POSSIBLE

YOUR WORK MUST SUPPORT YOUR ANSWER FOR FULL CREDIT TO BE AWARDED!

NO GRAPHING CALCULATOR AND NO DECIMALS

1. (18 POINTS) Use the graph of $y = f(x)$ shown below to answer the following questions. If it is a limit question, only consider finite limits. **If a limit does not exist, explain why**.



a. (2 POINTS) $\lim_{x\to 5} f(x) =$ 4

b. (2 POINTS) $f(-9) =$ 7

c. (2 POINTS) $\lim_{x\to 0} f(x) =$ 1

d. (2 POINTS) $f(-2) =$ -5.2

e. (2 POINTS) $\lim_{x\to -1^+} f(x) =$ DNE

tends to ∞

f. (2 POINTS) $\lim_{x\to -1^-} f(x) =$ DNE

tends to -∞

g. (3 POINTS) Is this function continuous at $x = -5$?

Circle one: continuous at $x = -5$? not continuous at $x = -5$?

If x is not continuous at $x = -5$, state why citing one of the three conditions for continuity at a point that you learned from our **Calculus definition**.

h. (3 POINTS) Is this function continuous at $x = 5$?

Circle one: continuous at $x = 5$ not continuous at $x = 5$

If x is not continuous at $x = 5$, state why citing one of the three conditions for continuity at a point that you learned from our **Calculus definition**.

$\lim_{x\to 5} f(x) = 4 \neq f(5) = 6$ (Fails condition 3)

2. (6 POINTS) Find the limit L. Then find $\delta > 0$ such that $|f(x) - L| < 0.01$ whenever $0 < |x - c| < \delta$. Decimal approximation is okay.

$$\lim_{x \to 5}(3x+5) \overset{\text{D.S.}}{=} 3(5)+5 = 20$$

$f(x) = 3x+5$

$L = 20$

$\varepsilon = 0.01$

$c = 5$

$\delta = ?$

$|f(x) - L| < \varepsilon$

$|3x+5-20| < 0.01$

$|3x-15| < 0.01$

$|3(x-5)| < 0.01$

$|3||x-5| < 0.01$

$|x-5| < \frac{0.01}{3}$

$|x-5| < \frac{1}{300}$

$$\boxed{\delta = \frac{1}{300}}$$

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3. (6 POINTS) Consider the following table:

x	-1.5	-1.1	-1.01	-1.001	-1	-0.999	-0.99	-0.9	-0.5
$f(x)$	4	25	10000	1000000	?	1000000	10000	25	4

a. (4 POINTS/2 POINTS EACH) Determine whether $f(x)$ approaches $-\infty$ or ∞ as x approaches -1 from the left and from the right.

i. $\lim_{x \to -1^-} f(x) = \underline{\infty}$ ii. $\lim_{x \to -1^+} f(x) = \underline{\infty}$

b. (2 POINTS) The graph of $f(x)$ has a ____________________________ at $x = -1$.

Circle one: Hole Vertical Asymptote

4. (30 POINTS) Find the exact **FINITE LIMIT** analytically. If there is no finite limit, write DNE (does not exist). Be sure to correctly use limit notation. DO NOT USE YOUR CALCULATOR!

a. (6 POINTS)

$$\lim_{x\to 3}\frac{x^3-3x^2+2x-6}{x^2-9} \overset{D.S.}{=} \frac{0}{0}$$

Factor

$$\lim_{x\to 3}\frac{x^3-3x^2+2x-6}{x^2-9} = \lim_{x\to 3}\frac{x^2(x-3)+2(x-3)}{(x+3)(x-3)}$$

$$= \lim_{x\to 3}\frac{\cancel{(x-3)}(x^2+2)}{(x+3)\cancel{(x-3)}}$$

$$= \lim_{x\to 3}\frac{x^2+2}{x+3}$$

$$\overset{D.S.}{=} \frac{(3)^2+2}{(3)+3}$$

$$= \boxed{\frac{11}{6}}$$

b. (3 POINTS)

$$\lim_{x\to 16} x^{-3/4} \overset{D.S.}{=} 16^{-3/4}$$

$$= \frac{1}{\left(\sqrt[4]{16}\right)^3}$$

$$= \boxed{\frac{1}{8}}$$

c. (6 POINTS)

$$\lim_{\theta\to 0}\frac{(1-\cos 4\theta)}{(\theta)}\frac{4}{4} = 4\lim_{\theta\to 0}\frac{1-\cos 4\theta}{4\theta}$$

$$= 4(0) \quad \text{special trig. limit}$$

$$= \boxed{0}$$

d. (3 POINTS)

$$\lim_{x\to \pi/4}\begin{cases}\sec x, & x<\pi\\ \tan x, & x\geq \pi\end{cases} \overset{D.S.}{=} \sec\frac{\pi}{4}$$

$$= \boxed{\sqrt{2}}$$

e. (6 POINTS)

$$\lim_{x\to 4}\frac{\sqrt{x}-2}{x-4} \overset{D.S.}{=} \frac{0}{0}$$

rationalize numerator

$$\lim_{x\to 4}\frac{(\sqrt{x}-2)}{(x-4)}\frac{(\sqrt{x}+2)}{(\sqrt{x}+2)} = \lim_{x\to 4}\frac{(\sqrt{x})^2-(2)^2}{(x-4)(\sqrt{x}+2)}$$

$$= \lim_{x\to 4}\frac{\cancel{x-4}^{\,1}}{\cancel{(x-4)}_{\,1}(\sqrt{x}+2)}$$

$$= \lim_{x\to 4}\frac{1}{\sqrt{x}+2}$$

$$\overset{D.S.}{=} \frac{1}{\sqrt{4}+2}$$

$$= \boxed{\frac{1}{4}}$$

f. (6 POINTS)

$$\lim_{\Delta x\to 0}\frac{(x+\Delta x)^2-x^2}{\Delta x} \overset{D.S.}{=} \frac{0}{0}$$

Factor diff. of squares

$$\lim_{\Delta x\to 0}\frac{(x+\Delta x)^2-x^2}{\Delta x} = \lim_{\Delta x\to 0}\frac{[(x+\Delta x)+x][(x+\Delta x)-x]}{\Delta x}$$

$$= \lim_{\Delta x\to 0}\frac{(2x+\Delta x)\cancel{\Delta x}}{\cancel{\Delta x}}$$

$$= \lim_{\Delta x\to 0}(2x+\Delta x)$$

$$\overset{D.S.}{=} 2x+0$$

$$= \boxed{2x}$$

5. (8 POINTS) Use the **limit process** to find the derivative of f with respect to x of $f(x)=\dfrac{1}{1-x}$.

$$f'(x) = \lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$$

common denominator

$$f'(x) = \lim_{\Delta x\to 0}\frac{\frac{(1-x)\,1}{(1-x)(1-(x+\Delta x))} - \frac{1}{1-x}\cdot\frac{(1-x-\Delta x)}{(1-x-\Delta x)}}{\Delta x}$$

$$f'(x) = \lim_{\Delta x\to 0}\frac{1-x-(1-x-\Delta x)}{(1-x-\Delta x)(1-x)}\cdot\frac{1}{\Delta x}$$

$$f'(x) = \lim_{\Delta x\to 0}\frac{1-x-1+x+\Delta x}{(1-x-\Delta x)(1-x)}\cdot\frac{1}{\Delta x}$$

$$f'(x) = \lim_{\Delta x\to 0}\frac{\cancel{\Delta x}\,1}{(1-x-\Delta x)(1-x)}\cdot\frac{1}{\cancel{\Delta x}}$$

$$f'(x) = \lim_{\Delta x\to 0}\frac{1}{(1-x-\Delta x)(1-x)}$$

$$f'(x) \overset{D.S.}{=} \frac{1}{(1-x-0)(1-x)}$$

$$\boxed{f'(x) = \frac{1}{(1-x)^2}}$$

6. (8 POINTS) Consider the function $f(x)=\frac{x^3-64}{x^2-16}$.

a. (3 POINTS) Write a simpler function $g(x)$ that agrees with the function at all but one point.

$$f(x)=\frac{(\cancel{x-4})(x^2+4x+16)}{(x+4)(\cancel{x-4})},\quad x\neq 4$$

$$f(x)=\frac{x^2+4x+16}{x+4},\quad x\neq 4$$

$$\boxed{g(x)=\frac{x^2+4x+16}{x+4}}$$

b. (3 POINTS) Find the one-sided limit. It is acceptable to write a result of plus or minus infinity.

$$\lim_{x\to -4^+}\frac{x^3-64}{x^2-16} = \lim_{x\to -4^+}\frac{x^2+4x+16}{x+4} = \boxed{\infty}$$

tends to infinity

try $x=-3.99$

$$\frac{(-3.99)^2+4(3.99)+16}{-3.99+4}$$

$= 1596.01$

c. (1 POINT) At $x=4$ the graph of f has a __________________________.
PLEASE CIRCLE ONE OF THE TWO CHOICES BELOW:

VERTICAL ASYMPTOTE HOLE

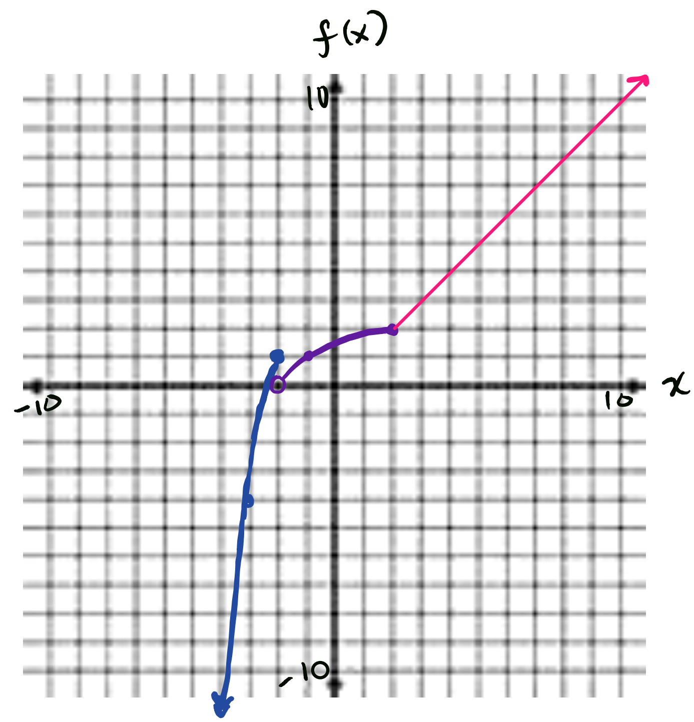
d. (1 POINT) At $x=-4$ the graph of f has a __________________________.
PLEASE CIRCLE ONE OF THE TWO CHOICES BELOW:

VERTICAL ASYMPTOTE HOLE

7. (12 POINTS) Consider the function

$$f(x)=\begin{cases} 5-x^2, & \text{if } x\le -2 \\ \sqrt{x+2}, & \text{if } -2<x\le 2 \\ x, & \text{if } x>2 \end{cases}$$

a. (6 POINTS) Sketch the graph. Be sure to label the graph and indicate the scale.



b. (3 POINTS) *The limit exists at all points on the graph* ***except*** *where:*

$\boxed{x=-2}$

c. (3 POINTS) On what interval(s) is this function continuous? Use interval notation.

$\boxed{(-\infty,-2)\cup(-2,\infty)}$

8. (12 POINTS, 3 POINTS EACH). Evaluate the following infinite limits below using the following information (it is okay to write a result of ∞ or $-\infty$:

$$\lim_{x\to c} f(x)=\infty,\ \lim_{x\to c} g(x)=-\frac{\sqrt{2}}{2},\ \text{and}\ \lim_{x\to c} h(x)=5$$

a. $\lim_{x\to c}\left[\frac{f(x)}{g(x)}\right] = \frac{\lim_{x\to c} f(x)}{\lim_{x\to c} g(x)} = \frac{\infty}{-\sqrt{2}/2} = \boxed{-\infty}$

b. $\lim_{x\to c}\left[\frac{-h(x)}{f(x)}\right] = \frac{-\lim_{x\to c} h(x)}{\lim_{x\to c} f(x)} = \frac{-5}{\infty} = \boxed{0}$

c. $\lim_{x\to c}\left(h(x)-[g(x)]^2\right) = \lim_{x\to c} h(x) - \left[\lim_{x\to c} g(x)\right]^2$

$= 5 - \left[-\sqrt{2}/2\right]^2$

$= 5 - \frac{1}{2}$

$= \boxed{\frac{9}{2}}$

d. $\sin^{-1}\left(\lim_{x\to c} g(x)\right) = \sin^{-1}\left(-\sqrt{2}/2\right)$

$= \boxed{-\frac{\pi}{4}}$