CALCULUS I/MATH 150

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EXAM 1/CHAPTERS 1, 2.1

- π 100 POINTS POSSIBLE
- π YOUR WORK MUST SUPPORT YOUR ANSWER FOR FULL CREDIT TO BE AWARDED
- π NO GRAPHING CALCULATOR IS PERMITTED
- π PROVIDE EXACT ANSWERS (NO DECIMALS PLEASE)

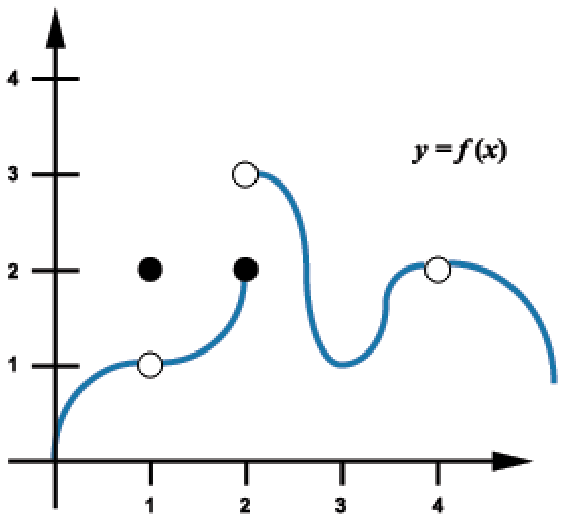
ONCE YOU BEGIN THE EXAM, YOU MAY NOT LEAVE THE PROCTORING CENTER UNTIL YOU ARE FINISHED. THIS MEANS NO BATHROOM BREAKS…

NAME Key

EXAM SCORE________________EXAM PERCENTAGE_______________

GOOD LUCK☺

1. (8 POINTS, 2 POINTS EACH) Use the graph of $y = f(x)$ shown below to find each limit, if it exists. **If the limit does not exist, explain why.**



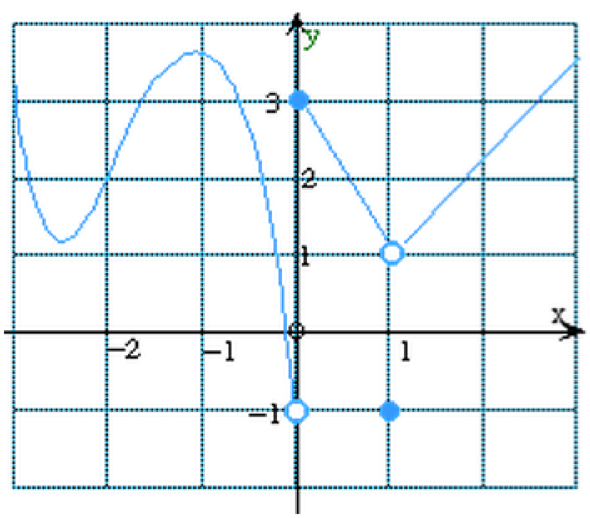
a. $\lim\limits_{x \to 2} f(x) =$ $\boxed{\text{DNE} \quad \overset{=2}{\lim_{x\to 2^-} f(x)} \neq \overset{=3}{\lim_{x\to 2^+} f(x)}}$

b. $\lim\limits_{x \to 1^-} f(x) =$ $\boxed{1}$

c. $\lim\limits_{x \to 1^+} f(x) =$ $\boxed{1}$

d. $\lim\limits_{x \to 1} f(x) =$ $\boxed{1}$

2. (6 POINTS) Consider the function shown below. Is this function continuous at $x = 0$? EXPLAIN using the **3 conditions** for continuity at a point!



1. $f(0) = 3$ ✓

2. $\lim\limits_{x \to 0} y$ DNE Fails

3.

Circle one:

continuous at $x = 1$ not continuous at $x = 1$

3. (6 POINTS) Find the limit L. Then find $\delta > 0$ such that $|f(x) - L| < 0.01$ whenever $0 < |x - c| < \delta$.

$\lim_{x\to 10}(10-5x)$ D.S. $= 10 - 5(10) = -40$

$L = -40$

$f(x) = 10 - 5x$

$\varepsilon = 0.01$

$c = 10$

$|f(x) - L| < 0.01$

$|10-5x-(-40)| < 0.01$

$|50-5x| < 0.01$

$|-5(x-10)| < 0.01$

$|-5||x-10| < 0.01$

$5|x-10| < 0.01$

$|x-10| < \frac{0.01}{5}$

$0 < x-10 < 0.0019$

$\boxed{\delta = 0.0019}$

$500 \overline{)1.000}$ $.002$

but $\delta = .002$ gives us $f(10.002) = 39.99$ and $-39.99-(-40) = 0.01$

4. (25 POINTS, 5 POINTS EACH) Find the exact **FINITE LIMIT** analytically. If there is no finite limit, write DNE (does not exist). DO NOT USE YOUR CALCULATOR!

a. $\lim_{x\to\pi}\left(\sec^3(9x/4)\right) = \sec^3\left(\lim_{x\to\pi}\frac{9x}{4}\right)$

D.S.

$= \left(\sec\frac{9\pi}{4}\right)^3$

$= (\sqrt{2})^3$

$= \boxed{2\sqrt{2}}$

b. $\lim_{x\to 4}\left(\frac{x^2-x-1}{x-1}\right)$ D.S. $= \frac{(4)^2-(4)-1}{(4)-1}$

$= \frac{11}{3}$

c. $\lim_{x\to 0}\frac{1-\cos 3x}{x}$ D.S. $= \frac{1-\cos(3\cdot 0)}{0}$

$= \frac{0}{0}$ indeterminate more work!

$\frac{3}{3}\lim_{x\to 0}\frac{1-\cos 3x}{x} = 3\lim_{x\to 0}\frac{1-\cos 3x}{3x}$

$= 3(0)$ ← special trig. limit

$= \boxed{0}$

d. $\lim_{x\to 1/3}\frac{3x-1}{27x^3-1}$ D.S. $= \frac{0}{0}$ indeterminate more work!

$\lim_{x\to\frac{1}{3}}\frac{3x-1}{27x^3-1} = \lim_{x\to\frac{1}{3}}\frac{\cancel{3x-1}}{\cancel{(3x-1)}(9x^2+3x+1)}$

$= \lim_{x\to\frac{1}{3}}\frac{1}{9x^2+3x+1}$

D.S. $= \frac{1}{9\left(\frac{1}{3}\right)^2+3\left(\frac{1}{3}\right)+1}$

$= \boxed{\frac{1}{3}}$

e. $\lim_{x\to -32}\left(-\sqrt[5]{x}+\sqrt[3]{2x}\right)$ D.S. $= -\sqrt[5]{-32}+\sqrt[3]{2(-32)}$

$= -(-2)+\sqrt[3]{-64}$

$= 2+(-4)$

$= \boxed{-2}$

5. (16 POINTS, 8 POINTS EACH) Find the exact **FINITE LIMIT** analytically. If there is no finite limit, write DNE (does not exist). DO NOT USE YOUR CALCULATOR!

a. $\lim_{x \to 2} \frac{\sqrt{x}-\sqrt{2}}{x-2} \overset{D.S.}{=} \frac{0}{0}$ indeterminate, more work!

$$\lim_{x \to 2} \frac{(\sqrt{x}-\sqrt{2})}{(x-2)} \frac{(\sqrt{x}+\sqrt{2})}{(\sqrt{x}+\sqrt{2})} = \lim_{x \to 2} \frac{(\sqrt{x})^2-(\sqrt{2})^2}{(x-2)(\sqrt{x}+\sqrt{2})}$$

$$= \lim_{x \to 2} \frac{\cancel{x-2}^{\,1}}{\cancel{(x-2)}_{\,1}(\sqrt{x}+\sqrt{2})}$$

$$= \lim_{x \to 2} \frac{1}{\sqrt{x}+\sqrt{2}} \overset{D.S.}{=} \frac{1}{\sqrt{2}+\sqrt{2}}$$

$$= \boxed{\frac{1}{2\sqrt{2}} \text{ or } \frac{\sqrt{2}}{4}}$$

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b. $\lim_{\Delta x \to 0} \frac{\frac{1}{x+\Delta x}-\frac{1}{x}}{\Delta x} \overset{D.S.}{=} \frac{0}{0}$ indeterminate, more work!

$$\lim_{\Delta x \to 0} \frac{\frac{1}{(x+\Delta x)}\frac{x}{x} - \frac{1}{x}\frac{(x+\Delta x)}{(x+\Delta x)}}{\Delta x} = \lim_{\Delta x \to 0} \frac{x-(x+\Delta x)}{x(x+\Delta x)} \cdot \frac{1}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-\cancel{\Delta x}^{\,1}}{x(x+\Delta x)} \cdot \frac{1}{\cancel{\Delta x}_{\,1}}$$

$$= \lim_{\Delta x \to 0} \frac{-1}{x(x+\Delta x)}$$

$$\overset{D.S.}{=} \frac{-1}{x(x+0)}$$

$$= \boxed{\frac{-1}{x^2}}$$

6. (10 POINTS) Use the limit definition to find the derivative of f with respect to x of $f(x) = \sin(x)$.

$$f'(x) = \lim_{\Delta x \to 0} \frac{\sin(x+\Delta x) - \sin(x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\sin x \cos \Delta x + \cos x \sin \Delta x - \sin x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cos x \sin \Delta x - \sin x (1 - \cos \Delta x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cos x \sin \Delta x}{\Delta x} - \lim_{\Delta x \to 0} \frac{\sin x (1 - \cos \Delta x)}{\Delta x}$$

$$= \cos x \lim_{\Delta x \to 0} \frac{\sin \Delta x}{\Delta x} - \sin x \lim_{\Delta x \to 0} \frac{1 - \cos \Delta x}{\Delta x}$$

$$= \cos x (1) - \sin x (0)$$

$$= \boxed{\cos x}$$

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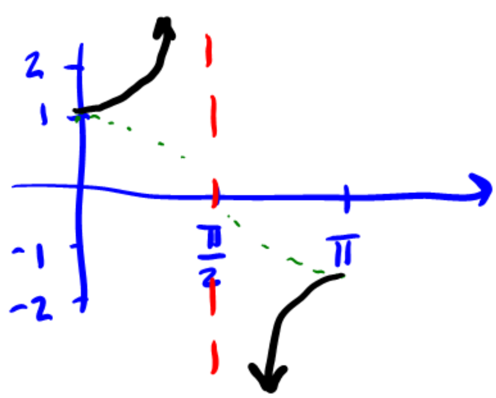
7. (7 POINTS) Find the limit. It is acceptable to write a result of plus or minus infinity.

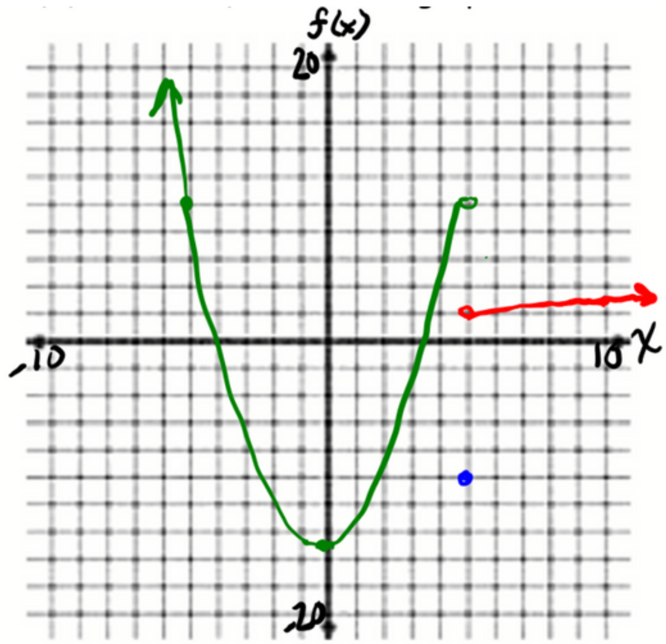
$$\lim_{x \to \pi/2^+} \sec x = \boxed{-\infty}$$



8. (10 POINTS) Consider the function

$$f(x)=\begin{cases} x^2-15, & \text{if } x<5 \\ -10, & \text{if } x=5 \\ \sqrt{x-1}, & \text{if } x>5 \end{cases}$$

a) (4 POINTS) Sketch the graph.



b) (3 POINTS) Identify the values of c, for which $\lim_{x \to c} f(x)$ exists. Use interval notation.

$\boxed{(-\infty,5)\cup(5,\infty)}$

c) (3 POINTS) On what interval(s) is this function continuous? Use interval notation.

$\boxed{(-\infty,5)\cup(5,\infty)}$

9. (12 POINTS, 3 POINTS EACH). Evaluate the limits below using the following information:

$$\lim_{x\to c} f(x)=\infty, \lim_{x\to c} g(x)=\frac{1}{2}, \text{ and } \lim_{x\to c} h(x)=5$$

a. $\lim_{x\to c}\left[\frac{h(x)}{f(x)}\right] = \frac{\lim_{x\to c} h(x)}{\lim_{x\to c} f(x)}$

$= \frac{5}{\infty}$

$= \boxed{0}$

b. $\lim_{x\to c}\left[g(x)f(x)\right]$

$= \left[\lim_{x\to c} g(x)\right]\left[\lim_{x\to c} f(x)\right]$

$= \left(\frac{1}{2}\right)(\infty)$

$= \boxed{\infty}$

c. $\lim_{x\to c}\left(-g(x)+\left[h(x)\right]^2\right)$

$= -\lim_{x\to c} g(x) + \left[\lim_{x\to c} h(x)\right]^2$

$= -\frac{1}{2} + (5)^2$

$= \boxed{\frac{49}{2}}$

d. $\cos^{-1}\left(\lim_{x\to c} g(x)\right) = \cos^{-1}\left(\frac{1}{2}\right)$

$= \boxed{\frac{\pi}{3}}$