When you finish your homework you should be able to…

π Curve sketch using methods from Calculus

Example 1: Sketch the graph of the equation by hand. If a particular characteristic of the graph does not occur, write "none".

$$f(x)=\frac{x}{x^2+1}$$

a. Intercepts (write as ordered pairs)

x-int: $0=\frac{x}{x^2+1} \to x=0$

y-int: $f(0)=0/1=0$

i. x-intercept: (0,0)

ii. y-intercept: (0,0)

b. Vertical Asymptote(s)

$x^2+1=0 \to$ no real zeros $\boxed{\text{NONE}}$

c. Behavior at vertical asymptote(s)

NONE

d. Horizontal Asymptote(s)

$$\lim_{x\to\infty}\frac{x/x^2}{\frac{x^2}{x^2}+\frac{1}{x^2}} = \lim_{x\to\infty}\frac{1/x}{1+\frac{1}{x^2}} = \frac{0}{1+0} = 0$$

$\boxed{y=0}$

$f(x) = \frac{x}{x^2+1}$

e. Run the test for increasing/decreasing intervals

$f'(x) = \frac{1(x^2+1) - x(\cancel{2x})}{(x^2+1)^2}$

$f'(x) = \frac{x^2+1-2x^2}{(x^2+1)^2}$

$f'(x) = \frac{1-x^2}{(x^2+1)^2}$

crit #:

$0 = 1 - x^2$

$\sqrt{x^2} = \sqrt{1}$

$x = \pm 1$

$(-\infty, -1) \mid (-1, 1) \mid (1, \infty)$

$- \quad + \quad -$

$-2 \quad -1 \quad 0 \quad 1 \quad 2$

$f'(x) = \frac{1-x^2}{(x^2+1)^2}$

$f'(-2) < 0$

$f'(0) > 0$

$f'(2) < 0$

i. f is increasing on $(-1, 1)$

ii. f is decreasing on $(-\infty, -1) \cup (1, \infty)$

f. Find the ordered pairs where relative extrema occur.

$f(-1) = \frac{-1}{(-1)^2+1} = -\frac{1}{2}$

$f(1) = \frac{1}{(1)^2+1} = \frac{1}{2}$

i. Relative minima: $(-1, -\frac{1}{2})$

ii. Relative maxima: $(1, \frac{1}{2})$

g. Test for concavity.

$$f'(x)=\frac{1-x^2}{(x^2+1)^2}$$

$$f''(x)=\frac{-2x(x^2+1)^{\cancel{2}}-(1-x^2)(2\cancel{(x^2+1)})(2x)}{(x^2+1)^{\cancel{4}\,3}}$$

$$f''(x)=\frac{-2x\left[(x^2+1)+2(1-x^2)\right]}{(x^2+1)^3}$$

$$f''(x)=\frac{-2x(x^2+1+2-2x^2)}{(x^2+1)^3}$$

$$f''(x)=\frac{-2x(3-x^2)}{(x^2+1)^3}$$

$0=-2x(3-x^2)$

$-2x=0$ or $3-x^2=0$

$x=0$ $\quad \sqrt{x^2}=\sqrt{3}$

$x=\pm\sqrt{3}$

$(-\infty,-\sqrt{3})$ $(-\sqrt{3},0)$ $(0,\sqrt{3})$ $(\sqrt{3},\infty)$

$-$ $\quad +$ $\quad -$ $\quad +$

-2 $\;-\sqrt{3}$ $\;-1$ $\;0$ $\;1$ $\;\sqrt{3}$ $\;2$

$$f''(x)=\frac{-2x(3-x^2)}{(x^2+1)^3}$$

$f''(-2)<0$ $\quad f''(1)<0$

$f''(-1)>0$ $\quad f''(2)>0$

i. f is concave upward on $(-\sqrt{3},0)\cup(\sqrt{3},\infty)$

ii. f is concave downward on $(-\infty,-\sqrt{3})\cup(0,\sqrt{3})$

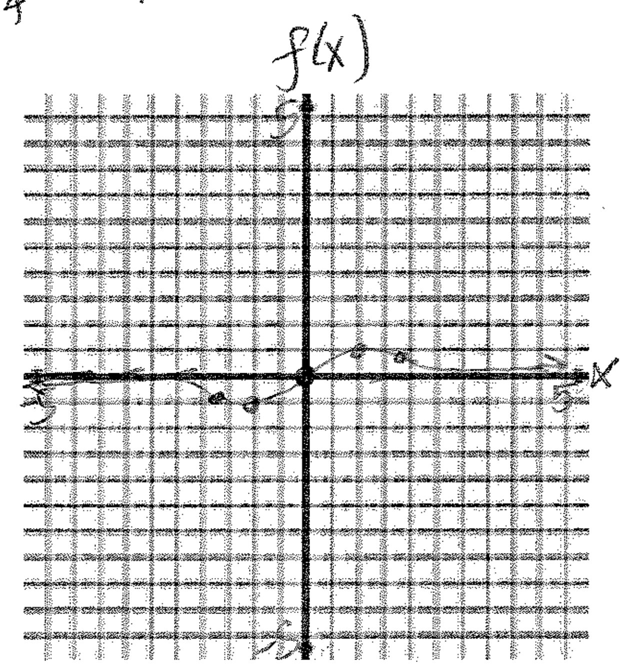
$\sqrt{3}\approx 1.73$

h. Find the points of inflection.

$f(x)=\frac{x}{x^2+1}$ $\qquad f(0)=0$

$f(-\sqrt{3})=\frac{-\sqrt{3}}{(-\sqrt{3})^2+1}=-\frac{\sqrt{3}}{4}\approx -0.43$ $\qquad f(\sqrt{3})=\frac{\sqrt{3}}{4}\approx 0.43$

i. Sketch the graph by hand.



Example 2: Sketch the graph of the equation by hand. If a particular characteristic of the graph does not occur, write "none".

$$f(x)=\frac{2x^2-5x+5}{x-2}$$

a. Intercepts (write as ordered pairs)

$f(0)=-\frac{5}{2}$ $0=2x^2-5x+5$ $b^2-4ac<0$

imaginary zeros

i. x-intercept: NONE

ii. y-intercept: $(0,-\frac{5}{2})$

b. Vertical Asymptote(s)

$$f(x)=\frac{2x^2-5x+5}{x-2}$$

$x-2=0$

$\boxed{x=2}$

c. Behavior at vertical asymptote(s)

$$\lim_{x\to 2^-}\frac{2x^2-5x+5}{x-2}=-\infty$$

$$\lim_{x\to 2^+}\frac{2x^2-5x+5}{x-2}=\infty$$

d. Horizontal Asymptote(s)

$$\lim_{x\to\infty}\frac{2x^2-5x+5}{x-2}=\infty\ ,\ \lim_{x\to-\infty}\frac{\frac{2x^2}{x}-\frac{5x}{x}+\frac{5}{x}}{\frac{x}{x}-\frac{2}{x}}=\lim_{x\to-\infty}\frac{2x-5+\frac{5}{x}}{1-\frac{2}{x}}=-\infty$$

e. Run the test for increasing/decreasing intervals

$$f(x)=\frac{2x^2-5x+5}{x-2}$$

$$f'(x)=\frac{(4x-5)(x-2)-(2x^2-5x+5)(1)}{(x-2)^2}$$

$$f'(x)=\frac{4x^2-13x+10-2x^2+5x-5}{(x-2)^2}$$

$$f'(x)=\frac{2x^2-8x+5}{(x-2)^2}$$

$$0=2x^2-8x+5$$

$$x=\frac{8\pm\sqrt{64-4(2)(5)}}{2(2)}$$

$$x=\frac{8\pm\sqrt{24}}{4}$$

$$x=\frac{8\pm2\sqrt{6}}{4}$$

$$x=\frac{\cancel{2}^{1}(4\pm\sqrt{6})}{\cancel{4}\,2}$$

$$x=\frac{4\pm\sqrt{6}}{2}$$

$x\approx 3.2$

$x\approx 0.8$

i. f is increasing on $(-\infty,0.8)\cup(3.2,\infty)$

ii. f is decreasing on $(0.8,2)\cup(2,3.2)$

$(-\infty,0.8)$ $(0.8,2)$ $(2,3.2)$ $(3.2,\infty)$

$+$ $-$ $-$ $+$

0 0.8 1 2 3 3.2 4

$f'(0)>0,\ f'(3)<0$

$f'(1)<0\quad f'(4)>0$

f. Find the ordered pairs where relative extrema occur.

rel. max at $(0.8, f(0.8))\rightarrow(0.8,-1.9)$

rel. min. at $(3.2, f(3.2))\rightarrow(3.2, 7.9)$

i. Relative minima: $(3.2, 7.9)$

ii. Relative maxima: $(0.8, -1.9)$

g. Test for concavity.

$$f'(x) = \frac{2x^2-8x+5}{(x-2)^2}$$

$$f''(x) = \frac{(4x-8)(x-2)^{\cancel{2}\,1} - (2x^2-8x+5)(2(x-2))(1)}{(x-2)^{\cancel{4}\,3}}$$

$$f''(x) = \frac{4x^2-16x+16-4x^2+16x-10}{(x-2)^3}$$

$$f''(x) = \frac{6}{(x-2)^3}$$

no zeros. $(-\infty,2)$ $(2,\infty)$

$-$ $+$

0 2 3

$f''(0)<0$, $f''(3)>0$

i. f is concave upward on $(2,\infty)$

ii. f is concave downward on $(-\infty,2)$

h. Find the points of inflection.

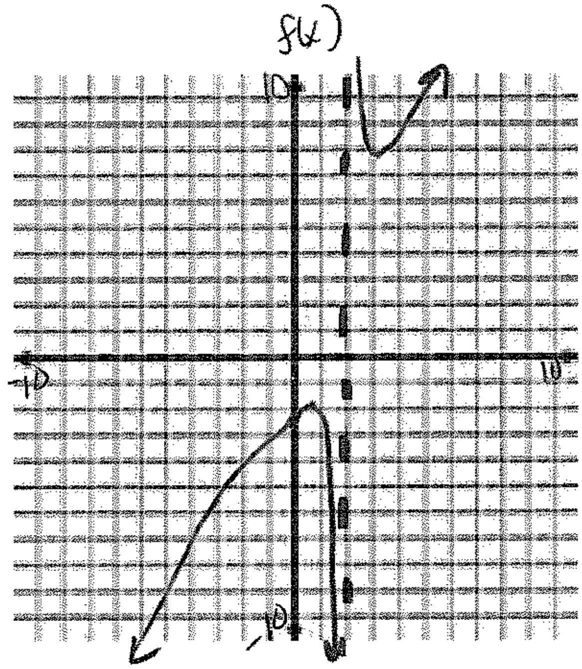
NONE

i. Sketch the graph by hand.

$$f(x) = \frac{2x^2-5x+5}{x-2}$$

$$f(-2) = \frac{8+10+5}{-4}$$

$$= -\frac{23}{4}$$



Example 3: Sketch the graph of the equation by hand. If a particular characteristic of the graph does not occur, write "none".

$f(x)=(x-1)^5$

a. Intercepts (write as ordered pairs)

$0=(x-1)^5 \quad f(0)=-1$

$0=x-1$

$1=x$

i. x-intercept: $(1,0)$

ii. y-intercept: $(0,-1)$

b. Vertical Asymptote(s)

$\boxed{\text{NONE}}$

c. Behavior at vertical asymptote(s)

$\boxed{\text{NONE}}$

d. Horizontal Asymptote(s)

$\lim_{x\to\infty}(x-1)^5=\infty$

$\lim_{x\to-\infty}(x-1)^5=-\infty$

NONE

e. Run the test for increasing/decreasing intervals

$f(x) = (x-1)^5$

$f'(x) = 5(x-1)^4(1)$

$0 = 5(x-1)^4$

$0 = (x-1)^4$

$0 = x-1$

$x = 1$

$(-\infty, 1) \mid (1, \infty)$

$+ \quad +$

$0 \quad 1 \quad 2$

$f'(x) = 5(x-1)^4$

$f'(0) > 0$

$f'(2) > 0$

i. f is increasing on $(-\infty,1) \cup (1,\infty)$

ii. f is decreasing on NONE

f. Find the ordered pairs where relative extrema occur.

$\emptyset$

i. Relative minima: NONE

ii. Relative maxima: NONE

g. Test for concavity.

$f'(x) = 5(x-1)^4$

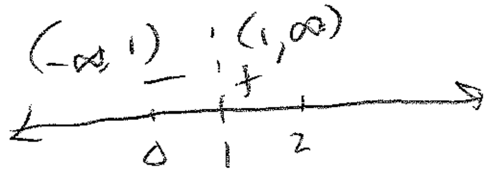
$f''(x) = 20(x-1)^3(1)$

$0 = 20(x-1)^3$

$0 = (x-1)^3$

$0 = x-1$

$x = 1$



$f''(x) = 20(x-1)^3$

$f''(0) < 0$

$f''(2) > 0$

i. f is concave upward on $(1, \infty)$

ii. f is concave downward on $(-\infty, 1)$

h. Find the points of inflection.

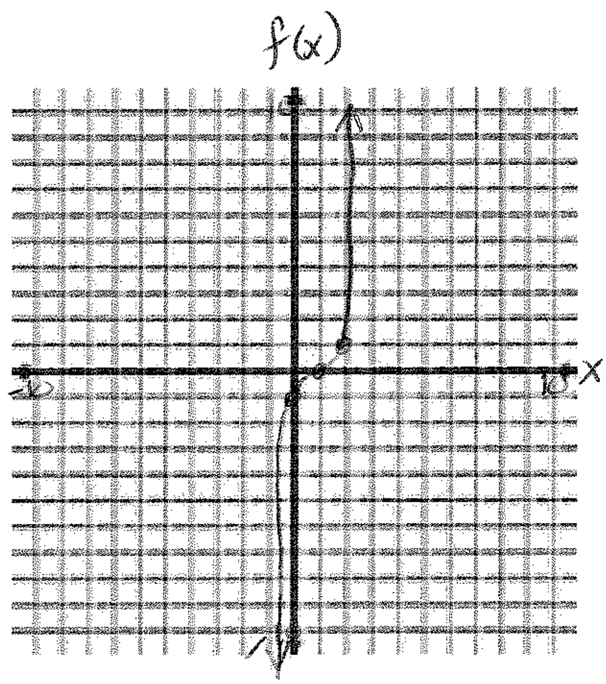
$f(1) = 0$

$\boxed{(1,0)}$

i. Sketch the graph by hand.

$f(x) = (x-1)^5$

$f(2) = 1$



Example 4: Sketch the graph of the equation by hand. If a particular characteristic of the graph does not occur, write "none".

$$f(x) = -x + 2\cos x,\ [0, 2\pi]$$

a. Intercepts (write as ordered pairs)

$0 = -x + 2\cos x$ | $f(0) = -0 + 2\cos 0 = 2$

$x \simeq 1.0299$

i. x-intercept: $(1.0299, 0)$

ii. y-intercept: $(0, 2)$

b. Vertical Asymptote(s)

NONE

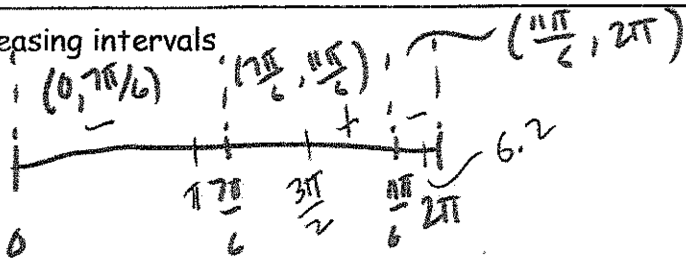
c. Behavior at vertical asymptote(s)

N/A

d. Horizontal Asymptote(s)

N/A $\rightarrow [0, 2\pi]$

e. Run the test for increasing/decreasing intervals



$f'(x) = -1 - 2\sin x$

$0 = -1 - 2\sin x$

$1 = -2\sin x$

$-\frac{1}{2} = \sin x$

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$

$f'(x) = -1 - 2\sin x$

$f'(\pi) = -1 - 0 < 0$

$f'(\frac{3\pi}{2}) = -1 + 2 > 0$

$f'(6.2) = -1 - 2(1.7) < 0$

i. f is increasing on $\left(\frac{7\pi}{6}, \frac{11\pi}{6}\right)$

ii. f is decreasing on $(0, 7\pi/6) \cup \left(\frac{11\pi}{6}, 2\pi\right)$

f. Find the ordered pairs where relative extrema occur.

$f\left(\frac{7\pi}{6}\right) = -\frac{7\pi}{6} + 2\cos\frac{7\pi}{6} = -\frac{7\pi}{6} - \sqrt{3} \approx -5.4$

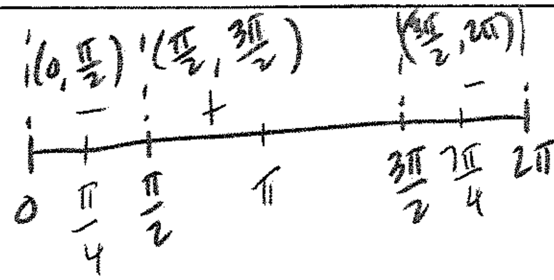
$\frac{7\pi}{6} \approx 3.7, \quad \frac{11\pi}{6} \approx 5.8$

$f\left(\frac{11\pi}{6}\right) = -\frac{11\pi}{6} + 2\cos\frac{11\pi}{6} = -\frac{11\pi}{6} + \sqrt{3} \approx -4.0$

i. Relative minima: $\left(\frac{7\pi}{6}, -\frac{7\pi}{6} - \sqrt{3}\right) \approx (3.7, -5.4)$

ii. Relative maxima: $\left(\frac{11\pi}{6}, -\frac{11\pi}{6} + \sqrt{3}\right) \approx (5.8, -4.0)$

g. Test for concavity.



$f'(x) = -1 - 2\sin x$

$f''(x) = -2\cos x$

$0 = -2\cos x$

$0 = \cos x$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

$f''(x) = -2\cos x$

$f''(\frac{\pi}{4}) = -\sqrt{2} < 0$ $\quad f''(\frac{7\pi}{4}) = -\sqrt{2}$

$f''(\pi) = 2 > 0$

i. f is concave upward on $(\frac{\pi}{2}, \frac{3\pi}{2})$

ii. f is concave downward on $(0, \frac{\pi}{2}) \cup (\frac{3\pi}{2}, 2\pi)$

h. Find the points of inflection.

$f(\frac{\pi}{2}) = -\frac{\pi}{2} + 2\cos\frac{\pi}{2} = -\frac{\pi}{2}$

$f(\frac{3\pi}{2}) = -\frac{3\pi}{2} + 2\cos\frac{3\pi}{2} = -\frac{3\pi}{2}$

$(\frac{\pi}{2}, -\frac{\pi}{2})$ and $(\frac{3\pi}{2}, -\frac{3\pi}{2})$

approx: $(1.6, -1.6)$ and $(4.7, -4.7)$

i. Sketch the graph by hand.

$f(0) = -0 + 2\cos 0 = 2$

$f(2\pi) = -2\pi + 2\cos 2\pi$

$= -2\pi + 2$

≈ -4.3