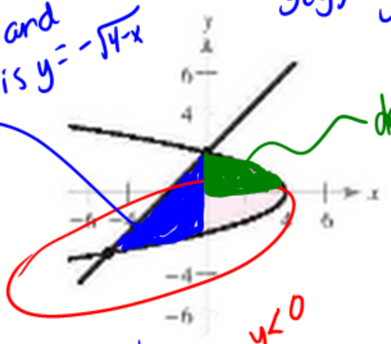
Review

7.1

17. $x = 4 - y^2 \rightarrow f(y) = 4 - y^2$

$x = y - 2 \rightarrow g(y) = y - 2$



① functions of y

Lim. of int:

$4 - y^2 = y - 2$

$0 = y^2 + y - 6$

$0 = (y+3)(y-2)$

$y = 2, -3$

Area

$A = \int_{-3}^{2} [(4-y^2) - (y-2)]\, dy$

$A = \int_{-3}^{2} (6 - y^2 - y)\, dy$

$A = \left(6y - \frac{y^3}{3} - \frac{y^2}{2}\right)\Big|_{-3}^{2}$

$A = \left[\left(12 - \frac{8}{3} - 2\right) - \left(-18 + 9 - \frac{9}{2}\right)\right]$

$A = \left[\left(10 - \frac{8}{3}\right) - \left(-9 - \frac{9}{2}\right)\right]$

$A = 19 - \frac{8}{3} + \frac{9}{2}$

$A = \frac{114 - 16 + 27}{6}$

$\boxed{A = \frac{125}{6} \text{ sq. units}}$

Announcements

- 5.6, 5.7, 7.1, 7.4 HW due Tues. 12/10
- EC due Tues. 12/10
- Final 9:30 - 11:30 Tues. 12/10
- Review Session 1:30 - 3:30 Rm OC3511 Friday 12/6

② functions of x

Lim. of int:

$y = 2: \; x = y - 2, \; x = 2 - 2 = 0$ / $y = -3: \; x = y - 2, \; x = -3 - 2 = -5$

Change to isolate y:

$x = 4 - y^2 \rightarrow y^2 = 4 - x$

$y = \pm\sqrt{4-x}$

(see picture above)

$x = y - 2 \rightarrow y = x + 2$

Area

blue region

green region

$A = \int_{-5}^{0} [(x+2) - (-\sqrt{4-x})]\, dx + 2\int_{0}^{4} \sqrt{4-x}\, dx$

$A = \left(\frac{x^2}{2} + 2x - \frac{2}{3}(4-x)^{3/2}\right)\Big|_{-5}^{0} - \frac{4}{3}(4-x)^{3/2}\Big|_{0}^{4}$

$A = \left[\left(0 + 0 - \frac{2}{3}(\sqrt{4})^3\right) - \left(\frac{25}{2} - 10 - \frac{2}{3}(\sqrt{9})^3\right)\right]$

$-\frac{4}{3}\left(0 - (\sqrt{4})^3\right)$

$A = -\frac{16}{3} - \left(\frac{5}{2} - \frac{2}{3} \cdot 27\right) - \frac{4}{3}(-8)$

$$A = -\frac{16}{3} + \frac{32}{3} - \frac{5}{2} + \frac{54}{3}$$

$$A = \frac{70}{3} - \frac{5}{2}$$

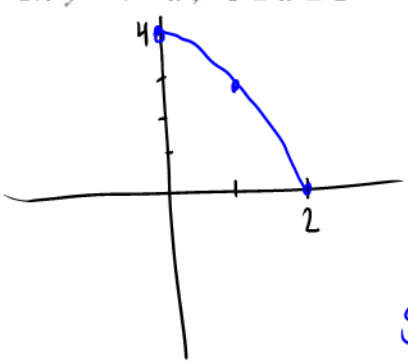
$$A = \frac{140-15}{6}$$

$$\boxed{A = \frac{125}{6} \text{ sq. units}}$$

7.4

In Exercises 17–26, (a) sketch the graph of the function, highlighting the part indicated by the given interval, (b) find a definite integral that represents the arc length of the curve over the indicated interval and observe that the integral cannot be evaluated with the techniques studied so far, and (c) use the integration capabilities of a graphing utility to approximate the arc length.

17. $y = 4 - x^2, \quad 0 \le x \le 2$



$$y' = -2x$$

$$1+(y')^2 = 1+4x^2$$

$$s = \int_a^b \sqrt{1+(y')^2}\, dx$$

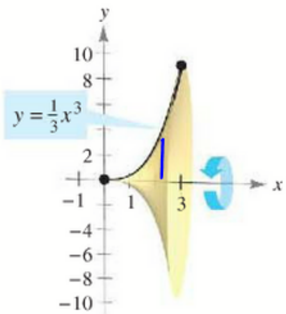
$$s = \int_0^2 \sqrt{1+4x^2}\, dx$$

requires a technique called trig. substitution (8.3)

7.4

In Exercises 37–42, set up and evaluate the definite integral for the area of the surface generated by revolving the curve about the x-axis.

37. $y = \frac{1}{3}x^3$



$$S = 2\pi\int_a^b r(x)\sqrt{1+(y')^2}\,dx$$ or

$$S = 2\pi\int_c^d r(y)\sqrt{1+(x')^2}\,dy$$

$y = \frac{1}{3}x^3$

$3y = x^3$

$\sqrt[3]{3y} = x$

$x = (3y)^{1/3}$

$x' = y^{-2/3}$

$$S = 2\pi\int_0^9 y\sqrt{1+(y^{-2/3})^2}\,dy$$

$$S = 2\pi\int_0^9 y\sqrt{1+\frac{1}{y^{4/3}}}\,dy$$

$$S = 2\pi\int_0^9 y\sqrt{\frac{y^{4/3}+1}{y^{4/3}}}\,dy$$

$$S = 2\pi\int_0^9 \frac{y}{y^{2/3}}\sqrt{y^{4/3}+1}\,dy$$

$$S = \frac{3}{4}\,2\pi\int_0^9 \frac{4}{3}y^{1/3}\sqrt{y^{4/3}+1}\,dy$$

$$S = \frac{3\pi}{2}\cdot\frac{2}{3}\left(y^{4/3}+1\right)^{3/2}\Big|_0^9$$

$$S = \pi\left[\left(9^{4/3}+1\right)^{3/2}-1\right] \text{ sq. units}$$

Function of y HARDER!!

see below for function of x

$r(x) = y = \frac{1}{3}x^3$

$y' = x^2$

$1 + (y')^2 = 1 + x^4$

$$S = 2\pi \int_0^3 \frac{1}{3}x^3 \sqrt{1 + x^4}\, dx$$

$$S = \frac{2\pi}{3} \cdot \frac{1}{4} \int_0^3 4x^3 \sqrt{1+x^4}\, dx$$

$$S = \frac{\pi}{6} \cdot \frac{2}{3} (1 + x^4)^{3/2} \Big|_0^3$$

$$S = \frac{\pi}{9}\left[(1+81)^{3/2} - 1\right]$$

$$\boxed{S = \frac{\pi}{9}\left[(82)^{3/2} - 1\right]}$$