CALCULUS I/MATH 150
SHANNON GRACEY

π 100 POINTS POSSIBLE
π YOUR WORK MUST SUPPORT YOUR ANSWER FOR FULL CREDIT TO BE AWARDED
π YOU MAY USE A TI-83/84/85/86 CALCULATOR
π PROVIDE EXACT ANSWERS UNLESS OTHERWISE INDICATED

ONCE YOU BEGIN THE EXAM, YOU MAY NOT LEAVE THE PROCTORING CENTER UNTIL YOU ARE FINISHED...THIS MEANS NO BATHROOM BREAKS!

NAME________________________
(64 POINTS) Problems 1-8. Evaluate the definite integrals and find the indefinite integrals: Each question is worth 8 points. EXACT ANSWERS ONLY!!

1. \[ \int_{2}^{6} |x - 3| \, dx \]

2. \[ \int \frac{2\theta^2}{\sin^2 \theta} \, d\theta \]
3. \[ \int \frac{x}{\sqrt{1-x}} \, dx \]

4. \[ \int (1 + x^2)^3 \, dx \]

5. \[ \int \cos^2 5x \, dx \]
6. \[
\int \left( \frac{4x + x^{3/4}}{x^{1/4}} \right) \, dx
\]

7. \[
\int_{3}^{5} \frac{x^3 + 1}{x + 1} \, dx
\]

8. \[
\int_{\pi/4}^{\pi/3} \tan^3 x \sec^2 x \, dx
\]
9. (5 POINTS) Find the average value of the function \( f(x) = \frac{4}{x^2} \) on the interval \([1, 4]\).

10. (5 POINTS) Sketch the region whose area is given by the definite integral. Then use a geometric formula to evaluate the integral. \( \int_{0}^{2} 3x \, dx \).

11. (6 POINTS) Use differentials to approximate the value of the expression \( \sqrt{64.5} \).
12. (10 POINTS) Evaluate the definite integral by the limit definition.
\[
\int_{1}^{3} (x^2) \, dx
\]

13. (10 POINTS) From a thin piece of cardboard 10 in. by 10 in., square corners are cut out so that the sides can be folded up to make a box. What dimensions will yield a box of maximum volume? You must use calculus to solve; include your analysis, optimization, and verification—no credit awarded for trial and error! Round to the nearest tenth, if necessary.
Theorem: Summation Formulas

1. \[ \sum_{i=1}^{n} c = cn \]
2. \[ \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \]
3. \[ \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \]
4. \[ \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4} \]