PRECALCULUS I/MATH 126 (2188)

SHANNON MYERS

- π 100 POINTS POSSIBLE
- π YOUR WORK MUST SUPPORT YOUR ANSWER FOR FULL CREDIT TO BE AWARDED
- π YOU MAY USE A SCIENTIFIC AND/OR A TI-83/84/85/86 CALCULATOR
- π PROVIDE EXACT ANSWERS UNLESS OTHERWISE INDICATED
- π PLEASE MAKE SURE YOU ARE TAKING THE CORRECT EXAM!

ONCE YOU BEGIN THE EXAM, YOU MAY NOT LEAVE THE PROCTORING CENTER UNTIL YOU ARE FINISHED…THIS MEANS NO BATHROOM BREAKS!

NAME Key

MATH 126/PRECALCULUS I/MYERS

NAME Key

EXAM 2/100 POINTS POSSIBLE

YOUR GRAPHING CALCULATOR SHOULD ONLY BE USED TO CHECK YOUR RESULTS. CREDIT WILL BE AWARDED BASED ON WORK SHOWN. THERE WILL BE NO CREDIT FOR GUESSING. PLEASE PRESENT YOUR WORK IN AN ORGANIZED, EASY TO READ FASHION.

1. (6 POINTS) The point at which a company's profits equal zero is called the company's **break-even point**. Let R represent a company's revenue, let C represent the company's costs, and let x represent the number of units produced and sold each day.

   $R(x) = 16x$ and $C(x) = 2.25x + 12{,}000$

   a. Find the firm's break-even point.

   Break even when $R(x) = C(x)$

   $16x = 2.25x + 12000$

   $13.75x = 12000$

   $x \approx 872.73$

   $= 873$

   $\boxed{\text{The company will break even once they've produced and sold 873 units.}}$

   b. Find the number of units the company must sell to earn a profit.

   Profit when $R(x) > C(x)$

   $16x > 2.25x + 12000$

   $13.75x > 12000$

   $x > 872$

   $\boxed{\text{The company must sell more than 872 units to make a profit.}}$

2. $f(x) = 5x^2 + 2x - 1$.

   a. (2 POINTS) Determine, without graphing, whether the given polynomial function has a maximum value or minimum value. Explain.

   $\boxed{\text{f has a minimum value because the leading coefficient is } 5 > 0 \text{, which means the parabola opens upward.}}$

   b. (4 POINTS) Find the minimum value or maximum value without using your graphing calculator. Do not use your graphing calculator and show all work.

   Vertex:

   $$h = -\frac{b}{2a} = -\frac{2}{2(5)} = -\frac{1}{5}$$

   $$K = f\left(-\frac{b}{2a}\right) = f\left(-\frac{1}{5}\right) = 5\left(-\frac{1}{5}\right)^2 + 2\left(-\frac{1}{5}\right) - 1 = \frac{1}{5} - \frac{2}{5} - \frac{5}{5} = -\frac{8}{5}$$

   $\boxed{\text{The minimum value is } -\frac{8}{5}\text{. It is found at } \left(-\frac{1}{5}, -\frac{8}{5}\right).}$

3. (8 POINTS) Solve the following inequality. Do not use your graphing calculator and show all work.

$$\frac{2x+7}{x-10} \geq 1$$

Step 1: Get zero on one side and find the related function

$$\frac{2x+7}{x-10} \geq 1 \rightarrow \frac{2x+7}{x-10} - 1 \geq 0$$

Let $f(x) = \frac{2x+7}{x-10} - 1\,\frac{(x-10)}{(x-10)}$

$$f(x) = \frac{2x+7-x+10}{x-10}$$

$$f(x) = \frac{x+17}{x-10}$$

Step 2: Find Critical numbers

$$0 = \frac{x+17}{x-10}$$

$$0 = x+17$$

$$x = -17$$

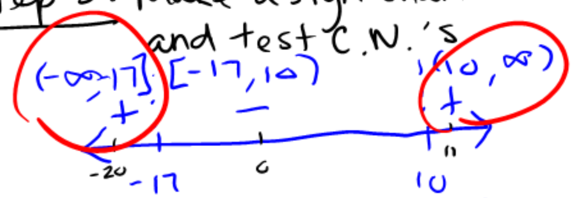
So $c_1 = -17$.

Also, f has a V.A. at $x-10=0$

$$x = 10$$

$$c_2 = 10$$

Step 3: Make a sign chart and test C.N.'s



$$f(-20) = \frac{-20+17}{-20-10} = \frac{-3}{-30} \rightarrow \text{positive}$$

$$f(0) = \frac{0+17}{0-10} = -\frac{17}{10} \rightarrow \text{negative}$$

$$f(11) = \frac{11+17}{11-10} = \frac{28}{1} \rightarrow \text{positive}$$

Step 4: Conclusion

$$\boxed{(-\infty,-17] \cup (10,\infty)}$$

4. (15 POINTS) Consider the quadratic function (4 POINTS) Solve $f(x) = -(x+3)^2 + 6$.

a. (4 POINTS) Find the zeros of f.

$0 = -(x+3)^2 + 6$

$\sqrt{(x+3)^2} = \pm\sqrt{6}$

$x + 3 = \pm\sqrt{6}$

$x = -3 \pm \sqrt{6}$

$\boxed{(-3-\sqrt{6}, -3+\sqrt{6})}$

Approx: $(-5.4, -0.6)$

b. (2 POINTS) Find the *y*-intercept.

$f(0) = -(0+3)^2 + 6$

$f(0) = -3$

$\boxed{(0, -3)}$

c. (4 POINTS) Find the vertex.

$f(x) = -(x+3)^2 + 6$

$f(x) = -[x-(-3)]^2 + 6$

vertex: $\boxed{(-3, 6)}$

d. (2 POINTS) What is the axis of symmetry?

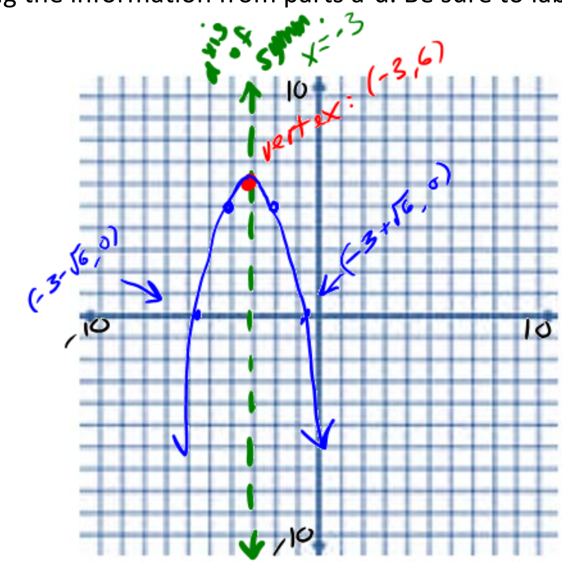
$x = h \rightarrow \boxed{x = -3}$

e. (3 POINTS) Sketch the graph of f **by hand** using the information from parts a-d. Be sure to label your graph.

$f(-2) = f(-4)$ by symmetry

$f(-2) = -[(-2)+3]^2 + 6$

$f(-2) = 5 = f(-4)$



5. (13 POINTS) Consider The function $f(x)=\dfrac{x^2-2x-8}{x^2-4}$.

a. (3 POINTS) If possible, simplify f.

$$f(x)=\frac{x^2-2x-8}{x^2-4}$$

$$f(x)=\frac{(x-4)\cancel{(x+2)}}{(x-2)\cancel{(x+2)}} \rightarrow \boxed{f(x)=\frac{x-4}{x-2},\ x\neq -2}$$

b. (4 POINTS) What is the domain of f ? Please describe how any domain restrictions appear on the graph of f.

Domain: $(-\infty,-2)\cup(-2,2)\cup(2,\infty)$

Restrictions:

At $x=-2$, there's a hole

At $x=2$, there a vertical asymptote

c. (2 POINTS) Find the vertical asymptote(s) of f. If there are no vertical asymptotes, write "none".

$x-2=0 \rightarrow \boxed{x=2}$

d. (2 POINTS) Find the horizontal asymptote(s) of f. If there are no horizontal asymptotes, write "none".

$$\begin{array}{r} 1 \\ (x-2)\overline{)\,x-4} \\ -(x-2) \\ \hline -2 \end{array}$$

As $x\rightarrow\infty$, the remainder $\rightarrow 0$

So $\boxed{y=1}$ is the HA.

e. (2 POINTS) Find the oblique (slant) asymptote of f. If there is no oblique asymptote, write "none".

$\boxed{\text{NONE}}$

6. (10 POINTS) Form a polynomial $f(x)$ with real coefficients having the given degree and zero.

Degree 4; zeros: $\sqrt{5},\ 2i$

$x = \pm\sqrt{5}$, $x = \pm 2i$

$x - \sqrt{5} = 0$, $x + \sqrt{5} = 0$, $x - 2i = 0$, $x + 2i = 0$

$0 = a(x-\sqrt{5})(x+\sqrt{5})(x-2i)(x+2i)$, where a is a constant.

$f(x) = 1(x-\sqrt{5})(x+\sqrt{5})(x-2i)(x+2i)$, let $a = 1$

$f(x) = [x^2 - (\sqrt{5})^2][x^2 - (2i)^2]$

$f(x) = (x^2 - 5)(x^2 - 4i^2)$

$f(x) = (x^2 - 5)[x^2 - 4(-1)]$

$f(x) = (x^2 - 5)(x^2 + 4)$

$\boxed{f(x) = x^4 - x^2 - 20}$

7. (12 POINTS) Consider the function $f\left(x\right)=2x^4-5x^3-x^2-5x-3$.

a. (2 POINTS) Use the rational zeros theorem to list the potential rational zeros of the polynomial function.

poss. for p: $\pm 1 \pm 3$

poss. for q: $\pm 1, \pm 2$

possibilities for $\frac{p}{q}$: $\frac{\pm 1, \pm 3}{\pm 1, \pm 2}$

$\boxed{\text{possible rational zeros: } \pm\frac{1}{2}, \pm 1, \pm\frac{3}{2}, \pm 3}$

b. (10 POINTS) Find the real zeros of f **without using your graphing calculator**. .

try $c=1$:

$$\begin{array}{r|rrrrr} 1 & 2 & -5 & -1 & -5 & -3 \\ & & 2 & -3 & -4 & -9 \\ \hline & 2 & -3 & -4 & -9 & -12 \end{array}$$

nope!

try $c=-1$:

$$\begin{array}{r|rrrrr} -1 & 2 & -5 & -1 & -5 & -3 \\ & & -2 & 7 & 6 & -1 \\ \hline & 2 & -7 & -6 & 1 & -4 \end{array}$$

nope!

try $c=-3$:

$$\begin{array}{r|rrrrr} -3 & 2 & -5 & -1 & -5 & -3 \\ & & -6 & 33 & & \\ \hline & 2 & -11 & & & \end{array}$$

try $c=\frac{1}{2}$:

$$\begin{array}{r|rrrrr} \frac{1}{2} & 2 & -5 & -1 & -5 & -3 \\ & & 1 & -2 & -\frac{3}{2} & \\ \hline & 2 & -4 & -3 & & \end{array}$$

try $c=-\frac{1}{2}$:

$$\begin{array}{r|rrrrr} -\frac{1}{2} & 2 & -5 & -1 & -5 & -3 \\ & & -1 & 3 & -1 & 3 \\ \hline & 2 & -6 & 2 & -6 & 0 \end{array}$$

yaay!!!...

So $c=\frac{1}{2}$ gives us:

$f(x)=2x^4-5x^3-x^2-5x-3$

$f(x)=(x-\frac{1}{2})(2x^3-6x^2+2x-6)$

$f(x)=(x-\frac{1}{2})\cdot 2(x^3-3x^2+x-3)$

$f(x)=(2x-1)[x^2(x-3)+1(x-3)]$

$f(x)=(2x-1)(x-3)(x^2+1)$

$2x-1=0 \rightarrow x=\frac{1}{2}$

$x-3=0 \rightarrow x=3$

$x^2+1=0 \rightarrow x=\pm\sqrt{-1}$ imaginary

$\boxed{\{\frac{1}{2}, 3\}}$

8. (30 POINTS)Consider the polynomial function $f(x)=(2x-3)^3(x-1)^2$

a. (2 POINTS) Determine the end behavior of the graph of the function:

The graph of f behaves like y = x^3 for large values of $|x|$.

b. (4 POINTS) Find the *x*-intercept(s), if any. Do not use your graphing calculator and show all work.

$(2x-3)^3=0$ or $(x-1)^2=0$

$2x-3=0$ $\qquad x-1=0$

$x=3/2$ $\qquad x=1$

$\boxed{\{1, \frac{3}{2}\}}$

c. (2 POINTS) Find the *y*-intercept, if any.

$f(0)=(2(0)-3)^3(0-1)^2=-27(1)=\boxed{-27}$

d. (9 POINTS) Determine the zeros of the function and their multiplicity. Use this information to determine whether the graph crosses or touches the *x*-axis.

i. The zero(s) of f are $\frac{3}{2}$ and 1.

ii. f has a zero of multiplicity 3, at $x=$ 3/2 so the graph of f crosses the *x*-axis at x = 3/2.

iii. f has a zero of multiplicity 2, at $x=$ 1 so the graph of f touches the *x*-axis at x = 1.

e. (3 POINTS) Use your graphing calculator to approximate the turning point(s) of the graph. Round the coordinates to 2 decimal places.

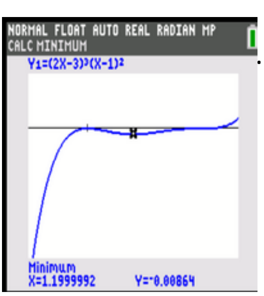
$\boxed{(1,0) \text{ and } (1.20,-0.01)}$

f. (2 POINTS) Find the domain of the function. You may use either interval or set-builder notation.

Domain: $\boxed{(-\infty,\infty) \text{ or } \{x \mid x \text{ is a real number}\}}$

g. (2 POINTS) Find the range of the function. You may use either interval or set-builder notation.

Range: $\boxed{(-\infty,\infty) \text{ or } \{y \mid y \text{ is a real number}\}}$



. (6 POINTS) Use the graph to determine where the function is increasing or decreasing. Give your results in interval notation.

i. On which interval(s) is the function increasing? Round the coordinates to 2 decimal places.

$\boxed{(-\infty,1)\cup(1.20,\infty)}$ or $\boxed{\{x \mid x\in\mathbb{R},\ x<1 \text{ or } x>1.20\}}$

ii. On which interval(s) is the function decreasing? Round the coordinates to 2 decimal places.

$\boxed{(1,\ 1.20)}$ or $\boxed{\{x \mid x\in\mathbb{R};\ 1<x<1.20\}}$