

Section 1.1: THE DISTANCE AND MIDPOINT FORMULAS; GRAPHING UTILITIES; INTRODUCTION TO GRAPHING EQUATIONS

When you are done with your homework you should be able to...

- π Use the Distance Formula
- π Use the Midpoint Formula
- π Graph Equations by Hand by Plotting Points
- π Graph Equations Using a Graphing Utility
- π Use a Graphing Utility to Create Tables
- π Find Intercepts from a Graph
- π Use a Graphing Utility to Approximate Intercepts

WARM-UP:

What grade do you want to earn in this class?

For each unit, how many hours should you spend on the class?

How many hours for "class time"?

How many hours for homework, test prep, etc.?

When should you work on math?

RECTANGULAR COORDINATES

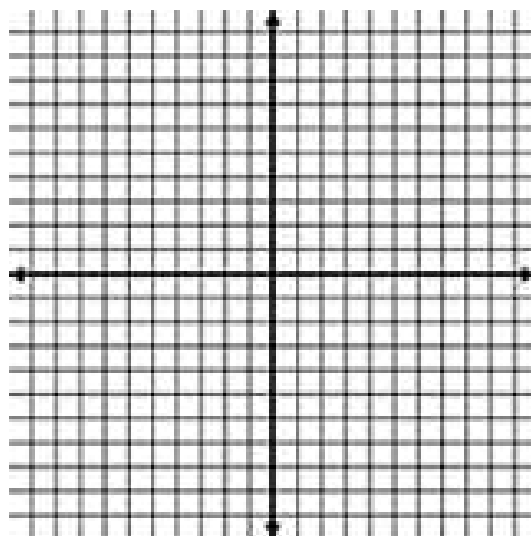
We locate a _____ on the real number line by assigning it a single real number, called the _____ of the point. For work in a two-dimensional _____, points are located by using _____ numbers.

The rectangular or Cartesian coordinate system consists of _____ real number lines, one _____ and one _____. The horizontal line is called the _____ and the vertical line is called the _____.

The point of intersection is located at the ordered pair _____ and is

called the _____. Assign _____ to every point on these number lines using a convenient scale. The scale of a number line is the distance between _____ and _____. Once you set the scale, it stays the same on that particular axis. Sometimes the scale on the x - and y -axes differ. For example, if you are sketching a line that has x -coordinates that can be easily viewed using a scale between -6 and 6 and y -coordinates that are better viewed between -1 and 1 , you may want to set the scale for the x -axis as _____ and the y -axis as _____.

Points on the x -axis to the right of O are associated with _____ real numbers, and those to the _____ of O are associated with _____ real numbers. Points on the y -axis above O are associated with _____ real numbers, and those _____ O are associated with _____ real numbers. The _____ divide the _____ into _____ regions, called _____. The points located on the _____ are _____ in any quadrant. Each _____ in the rectangular coordinate system _____ to an _____ of real numbers, _____.



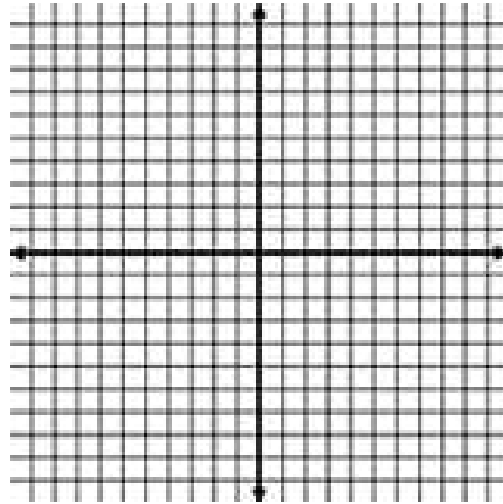
Example 1:

- a. Plot the following ordered pairs. Identify which quadrant or on what coordinate axis each point lies.

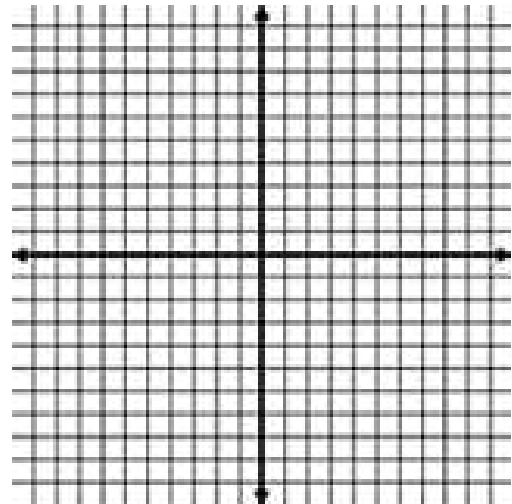
$$A = (2, 5)$$

$$B = (-3, 7)$$

$$C = (-2, 0)$$



- b. Plot the points $(0, 3)$, $(1, 3)$, $(5, 3)$, $(-4, 3)$. Describe the set of all points of the form $(x, 3)$ where x is a real number.



GRAPHING UTILITIES

All graphing utilities graph equations by _____ points. The screen itself consists of small rectangles, called _____. The more pixels the screen has, the better the resolution. When a point to be plotted lies inside a pixel, the pixel is turned on (lights up). The graph of an equation is a collection of _____ pixels.

The screen of a graphing calculator will display the coordinate axes of a rectangular coordinate system, but you need to set the _____ on each axis. You must also include the _____ and _____ values of _____ and _____ that you want included in the graph. This is called _____ the _____ or _____.

Xmin: the _____ value of _____ shown on the viewing window

Xmax: the _____ value of _____ shown on the viewing window

Xscl: the number of _____ per _____ mark on the _____

Ymin: the _____ value of _____ shown on the viewing window

Ymax: the _____ value of _____ shown on the viewing window

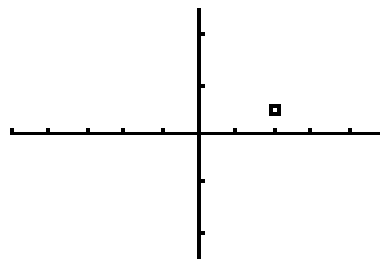
Yscl: the number of _____ per _____ mark on the _____

Example 2:

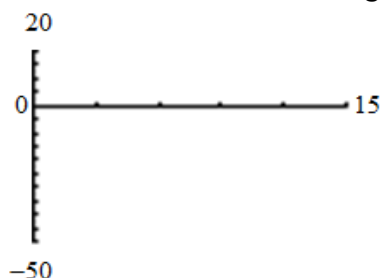
- Find the coordinates of the point shown below. Assume the coordinates are integers.

```

WINDOW
Xmin=-5
Xmax=5
Xscl=1
Ymin=-5
Ymax=5
Yscl=2
Xres=1
    
```



- Determine the viewing window used.



Xmin: _____

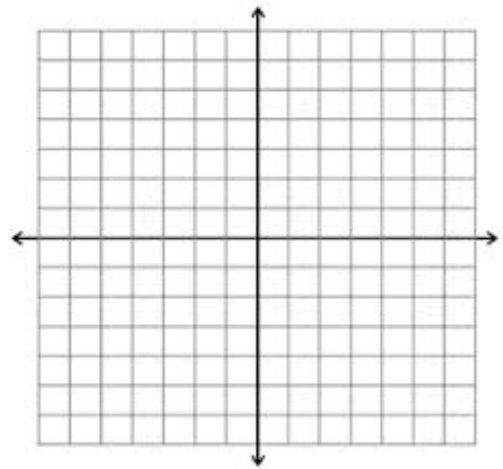
Xmax: _____

Xscl: _____

Ymin: _____

Ymax: _____

Yscl: _____



DISTANCE FORMULA

The distance between two points _____ and _____, denoted by _____, is

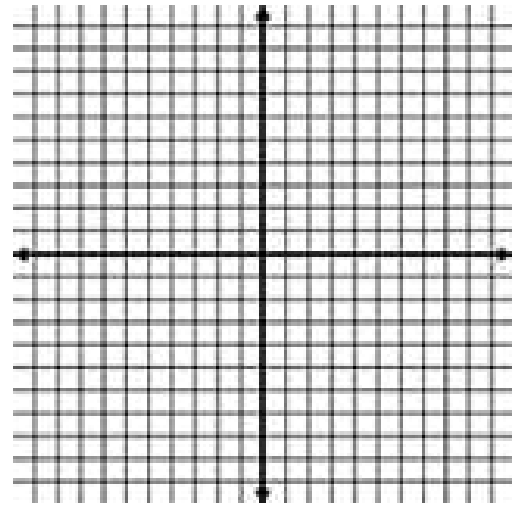
Example 3: Find the distance between each pair of points.

a. $P_1 = (-4, -3)$ and $P_2 = (6, 2)$

b. $P_1 = (a, a)$ and $P_2 = (0, 0)$

Example 4: Consider the points $A = (-2, 5)$, $B = (12, 3)$, and $C = (10, 11)$.

a. Plot each point and form the triangle ABC .



b. Verify that the triangle is a right triangle.

c. Find its area

THE MIDPOINT FORMULA

Consider a line segment whose endpoints are _____ and _____.

The midpoint, _____, is

Example 5: Find the midpoint of the line joining the points P_1 and P_2 .

a. $P_1 = (1,4)$ and $P_2 = (-2,7)$

b. $P_1 = (a,a)$ and $P_2 = (0,0)$

GRAPH EQUATIONS BY HAND BY PLOTTING POINTS

An _____ in _____, say _____ and _____, is a statement in which two expressions involving x and y are _____.

The expressions are called the _____ of the equation. Since an equation is a statement, it may be _____ or _____, depending on the value of the variables. Any values of x and y that result in a true statement are said to _____ the equation.

The _____ of an _____ in _____ x and y consists of the _____ of points in the _____ plane whose coordinates _____ satisfy the equation.

Example 6: Tell whether the given points are on the graph of the equation.

Equation: $y = x^3 - 2\sqrt{x}$

Points: $(0,0)$; $(1,1)$; $(1,-1)$

GRAPHING EQUATIONS USING A GRAPHING UTILITY

To graph an equation in two variables x and y using a graphing calculator requires that the dependent variable, y , be isolated.

PROCEDURES THAT RESULT IN EQUIVALENT EQUATIONS

1. Interchange the two sides of the equation:

_____ is equivalent to _____

2. Simplify the sides of the equation by combining like terms, eliminating parentheses, etc.:

_____ is equivalent to _____

3. Add or subtract the same expression on both sides of the equation:

_____ is equivalent to _____

4. Multiply or divide both sides of the equation by the same nonzero expression:

_____ is equivalent to _____

Example 7: Solve for y .

a. $5 - (x - 3) = 2y + 6\left(\frac{1}{2}y - 1\right)$

b. $4y - x^2 = 3$

HOW TO GRAPH AN EQUATION USING THE TI-83/TI-84 GRAPHING CALCULATOR

1. Solve the equation for _____ in terms of _____.
2. Enter the equation to be graphed into your graphing calculator.

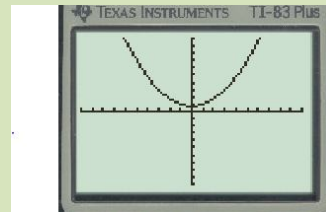
a. Use the "y =" key.



b. Graph the equation using the standard viewing window.

```
WINDOW
Xmin=-10
Xmax=10
Xscl=1
Ymin=-10
Ymax=10
Yscl=1
Xres=1
```

```
Plot1 Plot2 Plot3
Y1=(X^2+3)/4
Y2=
Y3=
Y4=
Y5=
Y6=
Y7=
```



c. Adjust the viewing window.

HOW TO USE THE TI-83/TI-84 TO CREATE TABLES

1. Solve the equation for _____ in terms of _____.
2. Enter the equation to be graphed into your graphing calculator.
3. Set up the table. In the AUTO mode, the user determines the starting point for the table and delta table, which determines the increment for x in the table. The ASK mode requires the user to enter values of x , and then the calculator determines the value of y .

```
Plot1 Plot2 Plot3
Y1=(X^2+3)/4
Y2=
Y3=
Y4=
Y5=
Y6=
Y7=
```



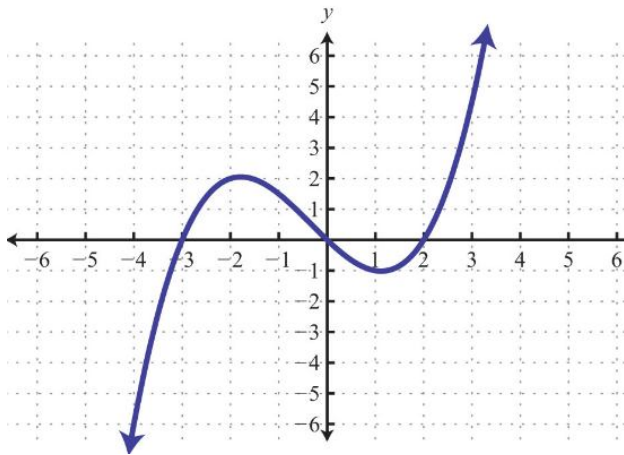
4. Create the table.

X	Y1
-3	3
-2	1.75
-1	1
0	.75
1	1
2	1.75
3	3

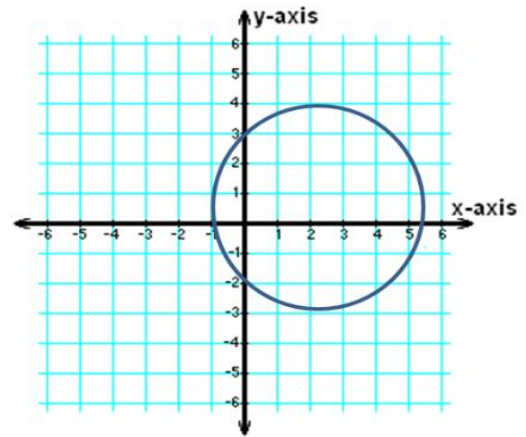
X=-3

Example 8: The graph of an equation is given. List the intercepts of the graph.

a.



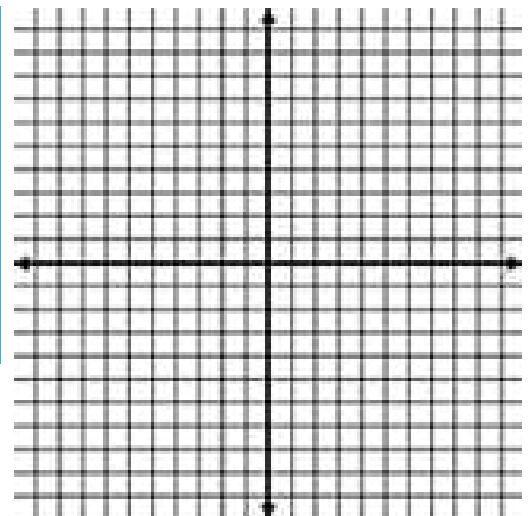
b.



Example 9: Graph each equation by hand by plotting points. Verify your results using a graphing utility.

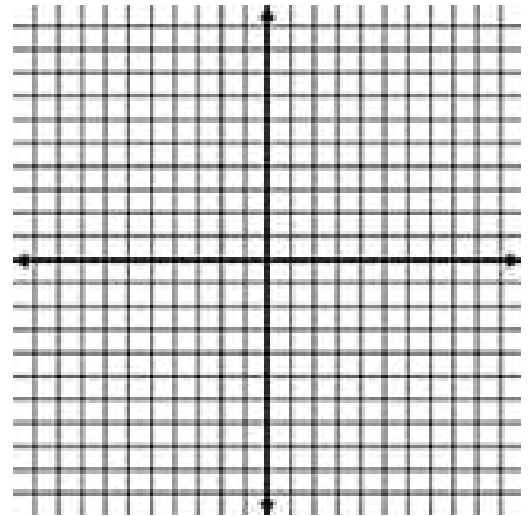
a. $y = 3x - 9$

x	$y = 3x - 9$	(x, y)



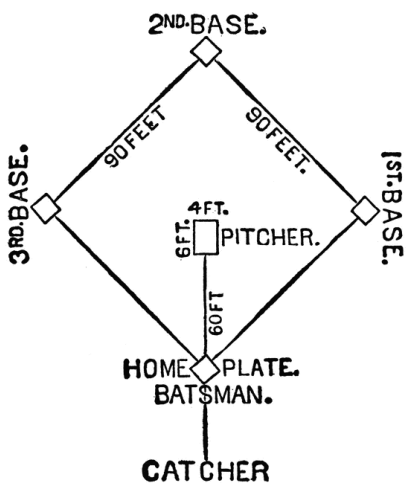
b. $y = -x^2 + 1$

x	$y = -x^2 + 1$	(x, y)



APPLICATIONS

A major league baseball “diamond” is actually a square, 90 feet on a side. What is the distance directly from home plate to second base (the diagonal of a square)? Give the exact simplified result first, and then round to the nearest hundredth.



Section 1.2: INTERCEPTS; SYMMETRY, GRAPHING KEY EQUATIONS

When you are done with your homework you should be able to...

π Find Intercepts Algebraically from an Equation

π Test an Equation for Symmetry

π Know How to Graph Key Equations

Warm-up: Solve.

a. $3x - 4(2x - 8) = 3 - 5x$

b. $2x^2 - x = 3$

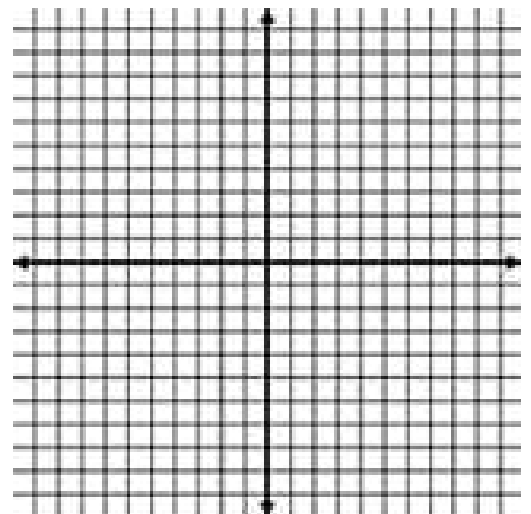
PROCEDURE FOR FINDING INTERCEPTS

1. To find the _____, if any, of the graph of an equation, let _____ in the equation and solve for _____, where _____ is a real number.
2. To find the _____, if any, of the graph of an equation, let _____ in the equation and solve for _____, where _____ is a real number.

Example 1: Find the intercepts and graph each equation by plotting points.

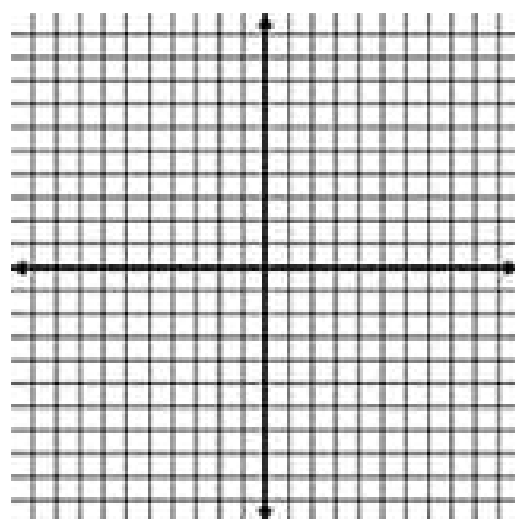
a. $y = x - 6$

x	$y = x - 6$	(x, y)



b. $4x^2 + y = 4$

x	$4x^2 + y = 4$	(x, y)

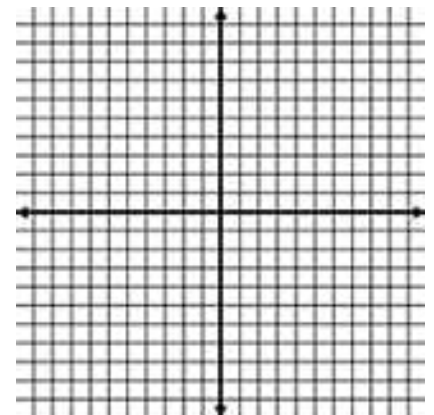
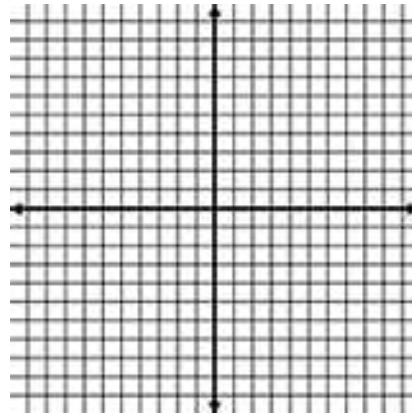
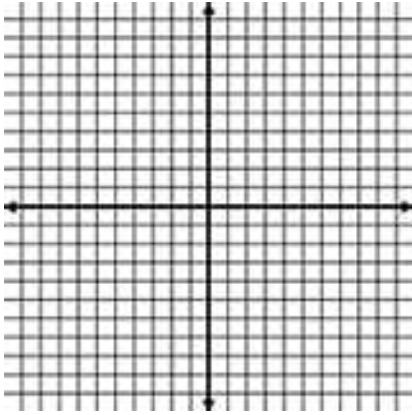


DEFINITION: SYMMETRY

A graph is said to be symmetric with respect to the _____ if, for every point _____ on the graph, the point _____ is also on the graph.

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A graph is said to be symmetric with respect to the _____ if, for every point _____ on the graph, the point _____ is also on the graph.



TESTS FOR SYMMETRY

_____ : Replace _____ by _____ in the equation. If an _____ equation results, the graph of the equation is _____ with respect to the _____.

_____ : Replace _____ by _____ in the equation. If an _____ equation results, the graph of the equation is _____ with respect to the _____.

_____ : Replace _____ by _____ and _____ by _____ in the equation. If an _____ equation results, the graph of the equation is _____ with respect to the _____.

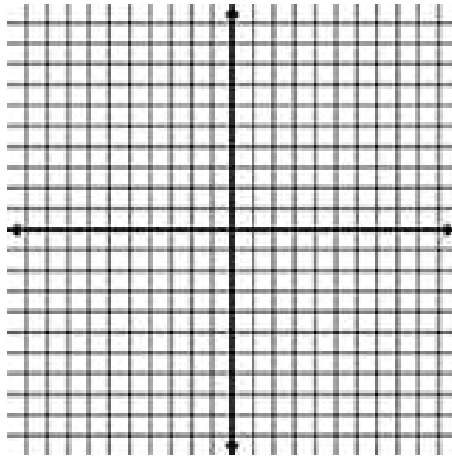
Example 2: Plot the point $(4, -2)$.

Plot the point that is symmetric to $(4, -2)$ with respect to the

a. x-axis

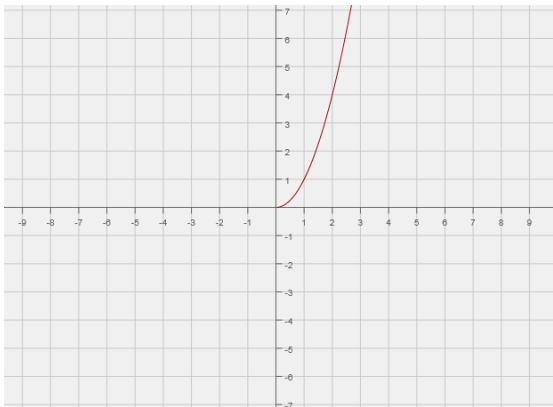
b. y-axis

c. origin

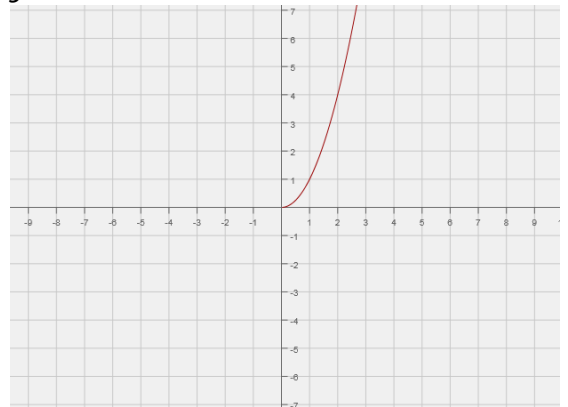


Example 3: Draw a complete graph so that it has the type of symmetry indicated.

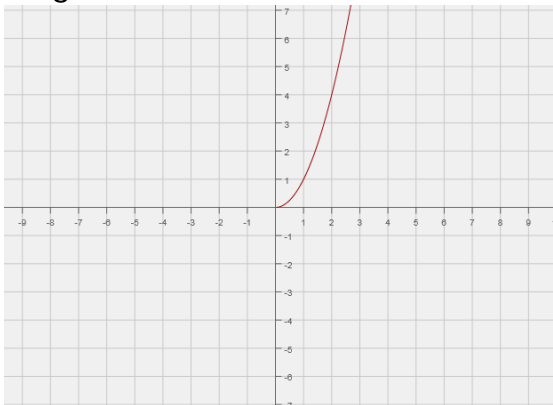
a. x-axis



b. y-axis



c. origin



Example 4: List the intercepts and test for symmetry.

a. $y^2 = x + 9$

b. $y = x^4 - 2x^2 - 8$

c. $y = \sqrt[5]{x}$

d. $y = \frac{x^4 + 1}{2x^5}$

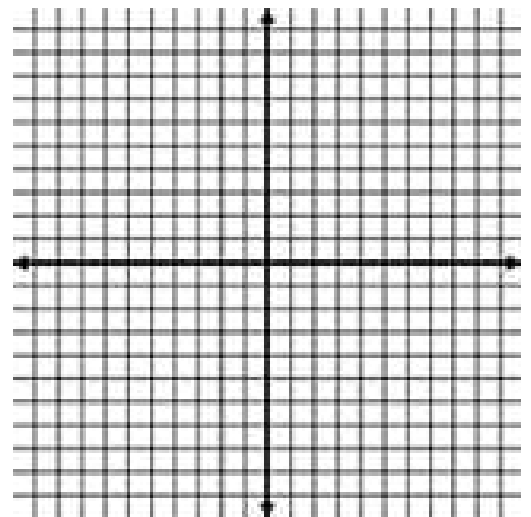
Example 5: If $(a, -5)$ is on the graph of $y = x^2 + 6x$, what is a ?

KNOW HOW TO GRAPH KEY EQUATIONS

Example 6: Sketch the graph using intercepts and symmetry.

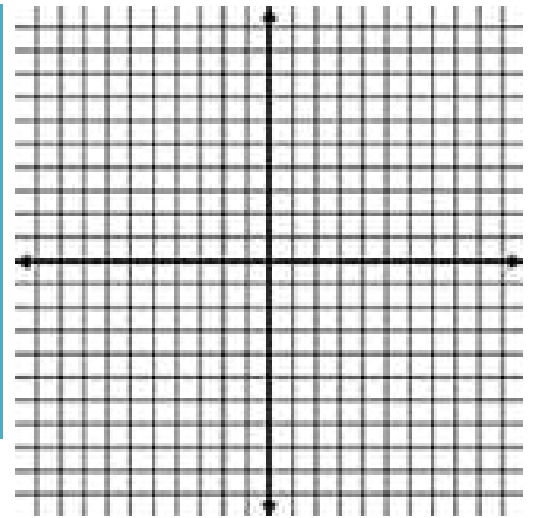
a. $y = \frac{1}{x}$

x	$y = \frac{1}{x}$	(x, y)



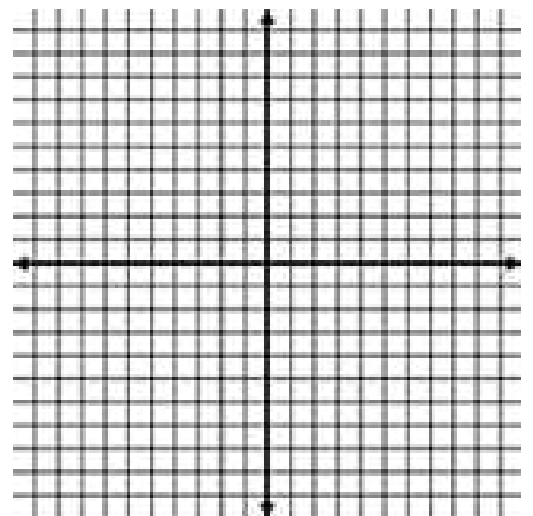
b. $x = y^2$

$x = y^2$	y	(x, y)



c. $y = x^3$

x	$y = x^3$	(x, y)



1.3: SOLVING EQUATIONS USING A GRAPHING UTILITY

When you are done with your homework, you should be able to...

π Solve Equations Using a TI -83/TI -84 Graphing Calculator

Warm-up: Solve for y .

a. $-x - 8y = 7$

b. $x^3 - 2y = 6$

SOLVE EQUATIONS USING A TI -83/TI -84 GRAPHING CALCULATOR

When a graphing calculator is used to solve an equation, usually

_____ solutions are obtained. Unless otherwise stated, we will approximate solutions as decimals rounded to _____ decimal places.

The _____ or _____ feature of a graphing calculator can be used to find the solutions of an equation when one side of the equation is _____. In using this feature to solve equations, we make use of the fact that when the graph of an equation in _____ variables, _____ and _____, crosses or touches the _____ then _____.

For this reason, any value of _____ for which _____ will be a _____ to the equation. That is, solving an equation for _____ when one side of the equation is 0 is equivalent to finding where the graph of the corresponding equation in two variables crosses or touches the _____.

STEPS FOR APPROXIMATING SOLUTIONS OF EQUATIONS USING ZERO OR ROOT

1. _____ y . So you will have $y = \{\text{expression in } x\}$.
2. Graph _____.
3. Use ZERO or ROOT to determine each _____ of the graph.

Example 1: Use ZERO or ROOT to approximate the real solutions, if any, of each equation rounded to two decimal places.

a. $-3x^4 + 8x^2 - 2x - 9 = 0$

b. $x^3 - 8x + 1 = 0$

STEPS FOR APPROXIMATING SOLUTIONS OF EQUATIONS USING INTERSECT

1. Graph _____ and graph _____
2. Use INTERSECT to determine the _____ of each point in the intersection.

Example 2: Use ZERO or ROOT to approximate the real solutions, if any, of each equation rounded to two decimal places.

a. $-x^4 + 1 = 2x^2 - 3$

b. $\frac{1}{4}x^3 - 5x = \frac{1}{5}x^2 - 4$

Example 3: Solve each equation algebraically. Verify your solution using a graphing calculator.

a. $5 - (2x - 1) = 10 - x$

b. $\frac{4}{y} - 5 = \frac{18}{2y}$

c. $x^3 + 2x^2 - 9x - 18 = 0$

1.4: LINES

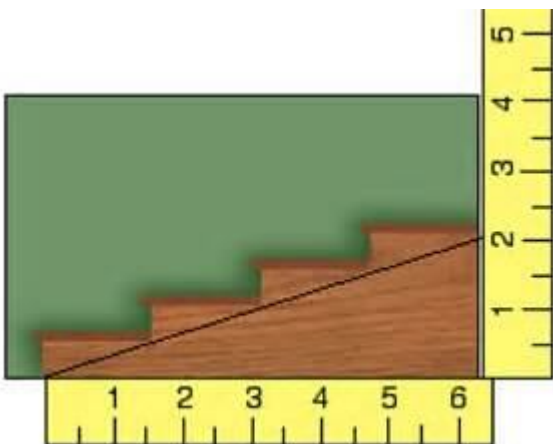
When you are done with your homework, you should be able to...

- π Calculate and Interpret the Slope of a Line
- π Graph Lines Given a Point and the Slope
- π Find the Equation of a Vertical Line
- π Use the Point-Slope Form of a Line; Identify Horizontal Lines
- π Find the Equation of a Line Given Two Points
- π Write the Equation of a Line in Slope Intercept Form
- π Identify the Slope and y-Intercept of a Line from Its Equation
- π Graph Lines Written in General Form Using Intercepts
- π Find Equations of Parallel Lines
- π Find Equations of Perpendicular Lines

Warm-up: Solve.

$$-5x + 2(1 - 3x) = x - 3$$

CALCULATE AND INTERPRET THE SLOPE OF A LINE



Consider the staircase to the left. Each step contains exactly the same horizontal _____ and the same vertical _____. The ratio of the rise to the run, called the _____, is a numerical measure of the _____ of the staircase.

DEFINITION

Let _____ and _____ be two distinct points. If _____, the _____, _____, of the nonvertical line L containing P and Q , is defined by the formula

If _____, L is a _____ line and the slope m of L is _____ (since this results in division by _____).

Example 1: Determine the slope of the line containing the given points.

a. $(4,2);(3,4)$

b. $(-1,1);(2,3)$

c. $(2,0);(2,2)$

SQUARE SCREENS

To get an undistorted view of slope, the same _____ must be used on each axis. Most graphing calculators have a rectangular screen. Because of this, using the same interval for x and y will result in a distorted view. On most graphing calculators, you can obtain a square screen by setting the ratio of x to y at _____.

Example 2: On the same square screen, graph the following equations:

$$y_1 = 0$$

$$y_2 = \frac{1}{2}x$$

$$y_3 = x$$

$$y_4 = 4x$$

Example 3: On the same square screen, graph the following equations:

$$y_1 = 0$$

$$y_2 = -\frac{1}{2}x$$

$$y_3 = -x$$

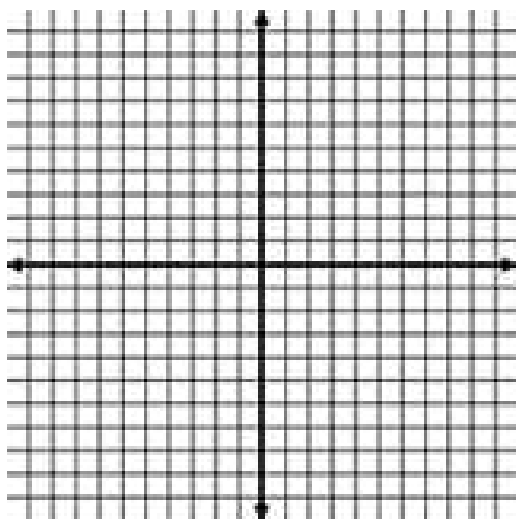
$$y_4 = -4x$$

What have we discovered???

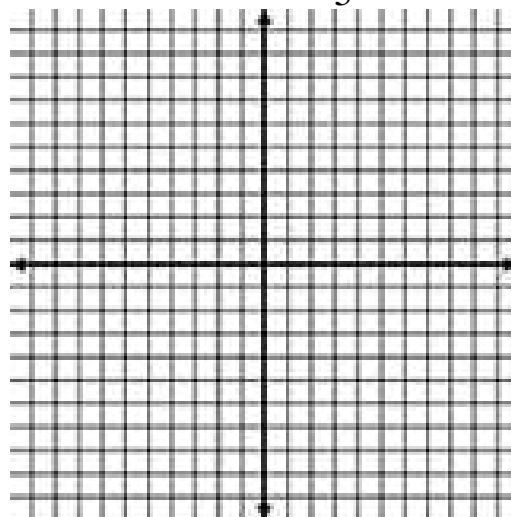
1. When the slope of a line is positive, the line slants _____
from left to right.
2. When the slope of a line is negative, the line slants _____
from left to right.
3. When the slope is zero, the line is _____.

Example 3: Graph the line containing the point P and having slope m . List two additional points that are on the line.

a. $P = (2,1); m = 4$



b. $P = (1,3); m = -\frac{2}{5}$



EQUATION OF A VERTICAL LINE

A vertical line is given by an equation of the form

where _____ is the _____.

POINT-SLOPE FORM OF A LINE

An equation of a nonvertical line with slope m that contains the point _____ is

EQUATION OF A HORIZONTAL LINE

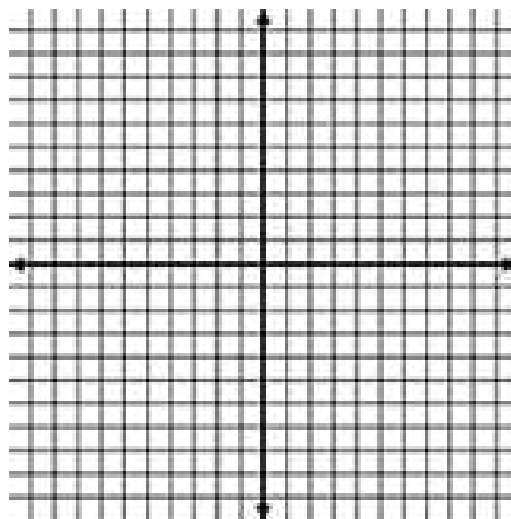
A horizontal line is given by an equation of the form

where _____ is the _____.

FINDING AN EQUATION OF A LINE GIVEN TWO POINTS

EXAMPLE 4: Find an equation of the line containing the points $(5, -1)$ and $(-6, 8)$.

Graph the line.



SLOPE-INTERCEPT FORM OF A LINE

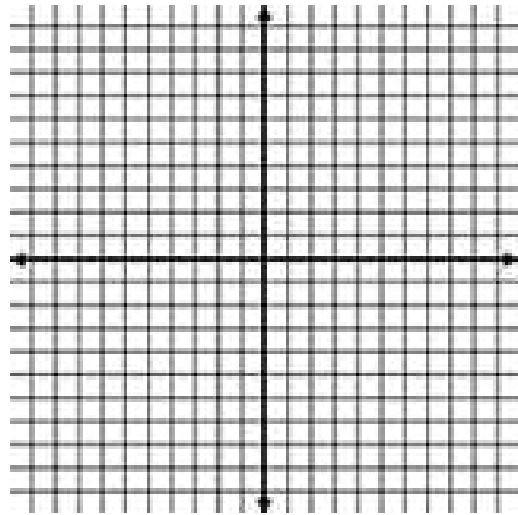
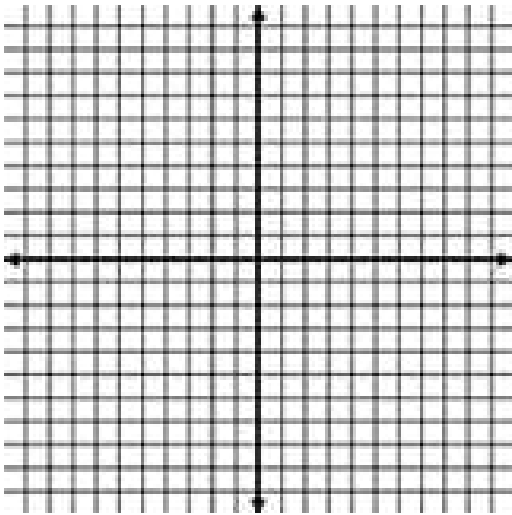
An equation of a line with slope m and y -intercept b is

IDENTIFY THE SLOPE AND y -INTERCEPT OF A LINE GIVEN ITS EQUATION

EXAMPLE 5: Find the slope m and y -intercept b of the given equation. Graph the equation.

a. $y = 5x + 2$

b. $2x - 3y = 9$



EQUATION OF A LINE IN GENERAL FORM

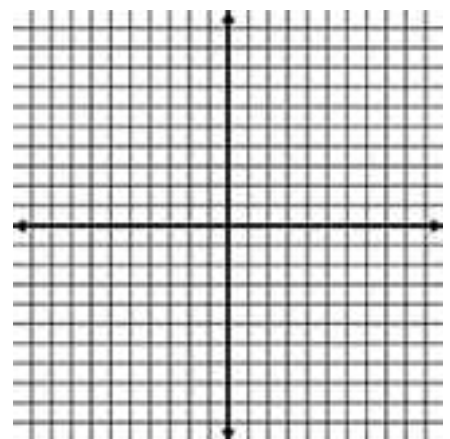
The equation of a line in general form is

where _____, _____, and _____ are real numbers and A and B are not both zero.

GRAPHING AN EQUATION IN GENERAL FORM USING ITS INTERCEPTS

EXAMPLE 6: Graph the equation $-4x + 2y = -12$ by finding its intercepts.

x	y	(x, y)



FIND EQUATIONS OF PARALLEL LINES

When two lines in the plane do not _____, they are said to be _____.

CRITERION FOR PARALLEL LINES

Two nonvertical lines are _____ if and only if their _____ are _____ and they have different _____.

SHOWING THAT TWO LINES ARE PARALLEL

EXAMPLE 7: Show that the lines given by the following equations are parallel.

$$y = -x + 4$$

$$x + y = -1$$

FIND EQUATIONS OF PERPENDICULAR LINES

When two lines _____ at a _____ angle _____, they are said to be _____.

CRI TERION FOR PERPENDI CULAR LINES

Two nonvertical lines are _____ if and only if the product of their _____ is _____.

SHOWING THAT TWO LINES ARE PERPENDICULAR

EXAMPLE 8: Show that the lines given by the following equations are perpendicular.

$$y = \frac{1}{2}x - 10$$

$$y = -2x - \frac{1}{3}$$

FINDING THE EQUATION OF A LINE GIVEN INFORMATION

The following examples will guide you on how to find the equation of a line when you are given different types of information.

EXAMPLE 9: Find an equation for the line with the given properties. Express your answer using the general form and the slope-intercept form.

- Slope = 2; containing the point $(4, -3)$.

b. Slope = undefined; containing the point $\left(\frac{1}{2}, 7\right)$.

c. x-intercept is -4; y-intercept is 4

d. Containing the points $(-3, 4)$ and $(2, 5)$.

e. Parallel to the line $x - 2y = -5$; containing the point $(-5, 1)$.

f. Perpendicular to the line $y = 8$; containing the point $(3, 4)$.

EXAMPLE 10: The equations of two lines are given. Determine if the lines are parallel, perpendicular, or neither.

a.

$$y = 4x + 5$$

$$y = -4x + 2$$

b.

$$y = \frac{1}{3}x - 3$$

$$y = -3x + 4$$

2.1: FUNCTIONS

When you are done with your homework, you should be able to...

- π Determine Whether a Relation Represents a Function
- π Find the Value of a Function
- π Find the Domain of a Function Defined by an Equation
- π Form the Sum, Difference, Product, and Quotient of Two Functions

WARM-UP: Find the value(s) of x for which the rational expression $\frac{x-1}{2x^2-x-10}$ is undefined.

DETERMINE WHETHER A RELATION REPRESENTS A FUNCTION

When the _____ of one variable is _____ to the value of a second variable, we have a _____. A relation is a _____ between two _____. If _____ and _____ are two elements in these sets and if a relation exists between _____ and _____, then we say that _____ to _____ or that _____ on _____, and we write _____.

Relations can be expressed as an _____, _____, and/or a _____.

Example 1: Find the domain and range of the relation.

VEHICLE	NUMBER OF WHEELS
CAR	4
MOTORCYCLE	2
BOAT	0

DEFINITION OF A FUNCTION

Let _____ and _____ represent two nonempty sets. A _____ from _____ into _____ is a relation that associates with each _____ of _____ exactly _____ element of _____.

FUNCTIONS AS EQUATIONS AND FUNCTION NOTATION

Functions are often given in terms of _____ rather than as _____ of _____. Consider the equation below, which describes the position of an object, in feet, dropped from a height of 500 feet after x seconds.

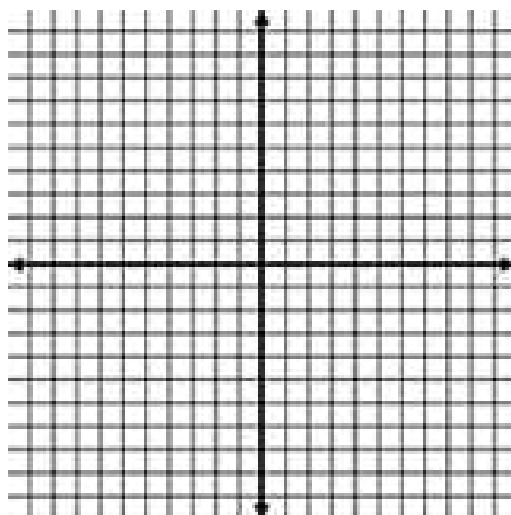
$$y = -16x^2 + 500$$

The variable _____ is a _____ of the variable _____. For each value of x , there is one and only one value of _____. The variable x is called the _____ variable because it can be _____ any value from the _____. The variable y is called the _____ variable because its value _____ on x . When an _____ represents a _____, the function is often named by a letter such as f , g , h , F , G , or H . Any letter can be used to name a function. The domain is

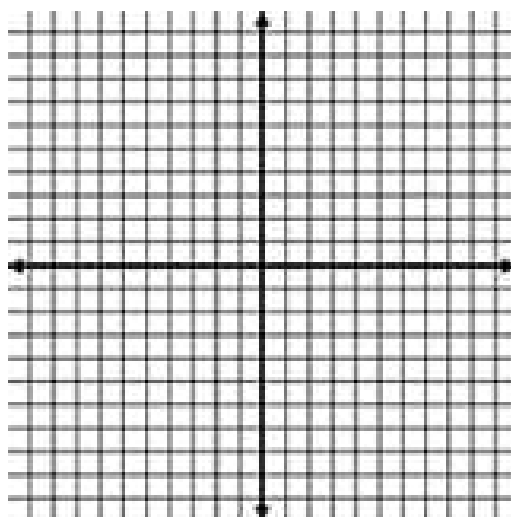
the _____ of the function's _____ and the range is the _____ of the function's _____. If we name our function _____, the input is represented by _____, and the output is represented by _____. The notation _____ is read " _____ of _____" or " _____ at _____". So we may rewrite $y = -16x^2 + 500$ as _____. Now let's evaluate our function after 1 second:

Example 2: Determine whether each relation represents a function. Then identify the domain and range.

a. $\{(-6,1), (-1,1), (0,1), (1,1), (2,1)\}$



b. $\{(3,3), (-2,0), (4,0), (-2,-5)\}$



Example 3: Find the indicated function values for $f(x) = -x^2 + 4x$.

a. $f(4)$

b. $3f(-2)$

c. $f(x+1)$

d. $\frac{f(x+h) - f(x)}{h}, h \neq 0$

Example 4: Find the indicated function and domain values using the table below.

a. $h(-2)$

b. $h(1)$

c. For what values of x is $h(x) = 1$?

x	$h(x)$
-2	2
-1	1
0	0
1	1
2	2

Example 5: Determine if the following equations define y as a function of x .

a. $xy = 5$

b. $x^2 + y^2 = 16$

FINDING VALUES OF A FUNCTION ON A CALCULATOR

Example 6: Let $f(x) = -x^3 - x + 2$. Use a graphing calculator to find the following values:

a. $f(4)$

b. $f(-2)$

STEPS FOR FINDING THE DOMAIN OF A FUNCTION DEFINED BY AN EQUATION

1. Start with the domain as the set of _____ numbers.
2. If the equation has a denominator, _____ any numbers that give a _____ denominator.
3. If the equation has a _____ of even _____, exclude any numbers that cause the expression inside the radical to be _____.

Example 7: Find the domain of each of the following functions.

a. $h(x) = \sqrt{2x-1}$

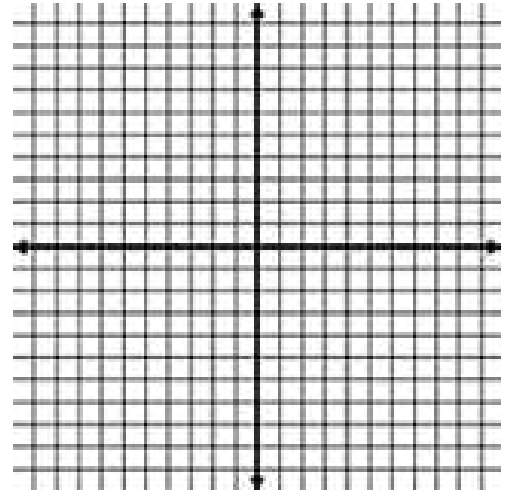
b. $g(x) = \frac{8x}{x^2 - 81}$

THE ALGEBRA OF FUNCTIONS

Consider the following two functions:

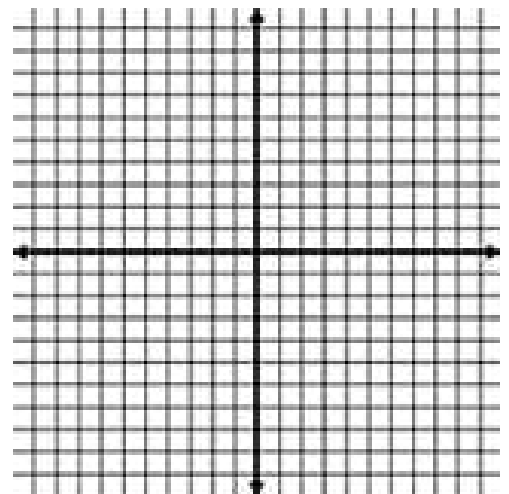
$$f(x) = 2x \text{ and } g(x) = x - 1$$

Let's graph these two functions on the same coordinate plane.



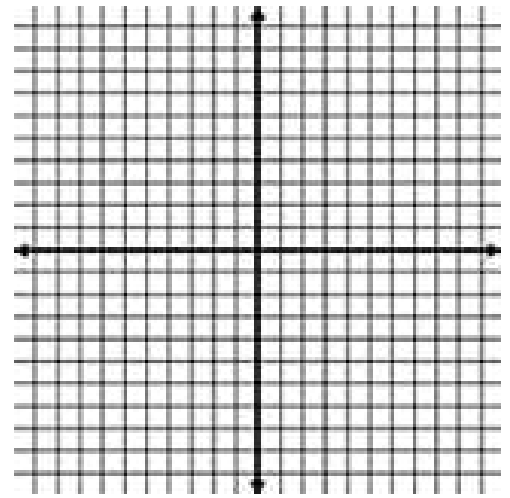
Now find and graph the sum of f and g .

$$(f + g)(x) =$$



Now find and graph the difference of f and g . $f(x) = 2x$ and $g(x) = x - 1$

$$(f - g)(x) =$$



Now find and graph the product of f and g on your graphing calculator.

$$(fg)(x) =$$

Now find and graph the quotient of f and g on your graphing calculator.

$$\left(\frac{f}{g}\right)(x) =$$

THE ALGEBRA OF FUNCTIONS: SUM, DIFFERENCE, PRODUCT, AND QUOTIENT OF FUNCTIONS

Let f and g be two functions. The _____ $f + g$, the _____ $f - g$, the _____ fg , and the _____ $\frac{f}{g}$ are _____ whose domains are the set of all real numbers _____ to the domains of f and g , defined as follows:

1. Sum: _____
2. Difference: _____
3. Product: _____
4. Quotient: _____, provided _____

Example 8: Let $f(x) = x^2 + 4x$ and $g(x) = 2 - x$. Find the following:

- a. $(f + g)(x)$
- b. $(f + g)(4)$
- c. $f(-3) + g(-3)$
- d. $(fg)(x)$
- e. $(fg)(3)$
- f. The domain of $\left(\frac{f}{g}\right)(x)$

2.2: THE GRAPH OF A FUNCTION

When you are done with your homework, you should be able to...

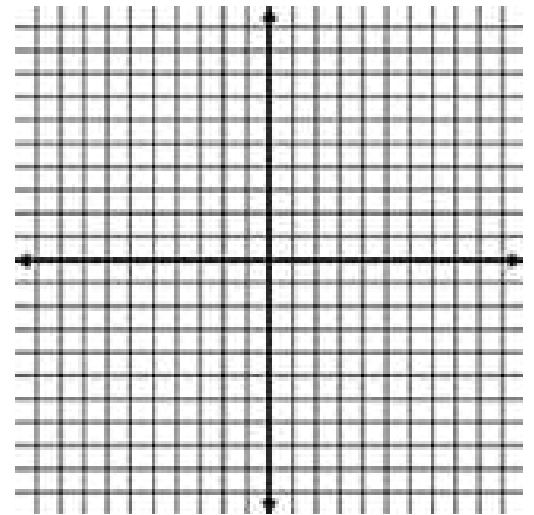
π Identify the Graph of a Function

π Obtain Information from or about the Graph of a Function

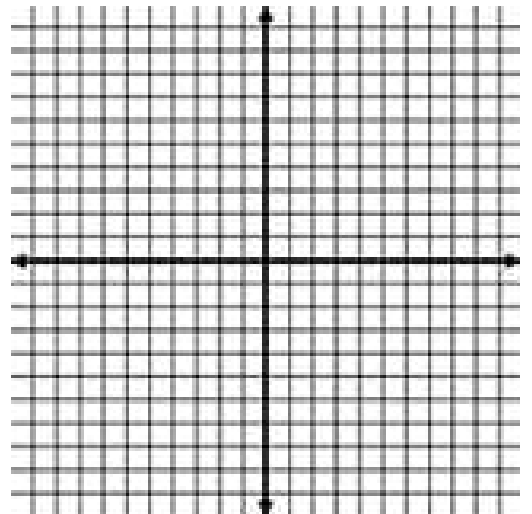
WARM-UP:

Graph the following equations by plotting points.

a. $y = x^2$



b. $y = 3x - 1$

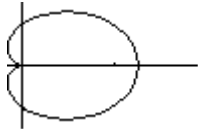


THE VERTICAL LINE TEST FOR FUNCTIONS

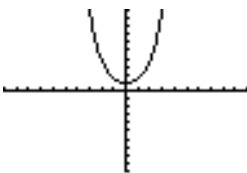
If any vertical line _____ a graph in more than _____ point, the graph _____ define _____ as a function of _____.

Example 1: Determine whether the graph is that of a function.

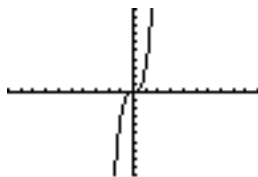
a.



b.



c.



OBTAINING INFORMATION FROM GRAPHS

You can obtain information about a function from its graph. At the right or left of a graph, you will often find _____ dots, _____ dots, or _____.

π A closed dot indicates that the graph does not _____ beyond this point and the _____ belongs to the _____

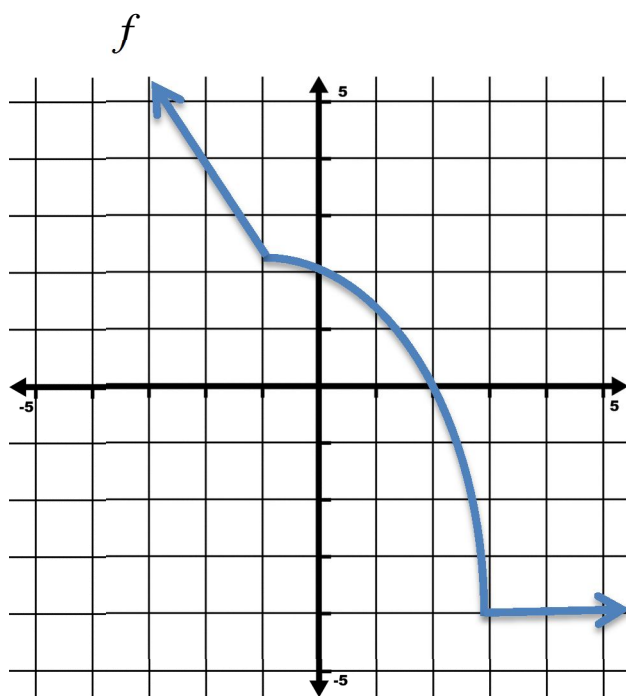
π An open dot indicates that the graph does not _____ beyond this point and the _____ DOES NOT belong to the _____

π An arrow indicates that the graph extends _____ in the direction in which the arrow _____

REVIEWING INTERVAL NOTATION

INTERVAL NOTATION	SET-BUILDER NOTATION	GRAPH
(a, b)		
$[a, b]$		
$[a, b)$		
$(a, b]$		
(a, ∞)		
$[a, \infty)$		
$(-\infty, b)$		
$(-\infty, b]$		
$(-\infty, \infty)$		

Example 2: Use the graph of f to determine each of the following.



a. $f(0)$

b. $f(-2)$

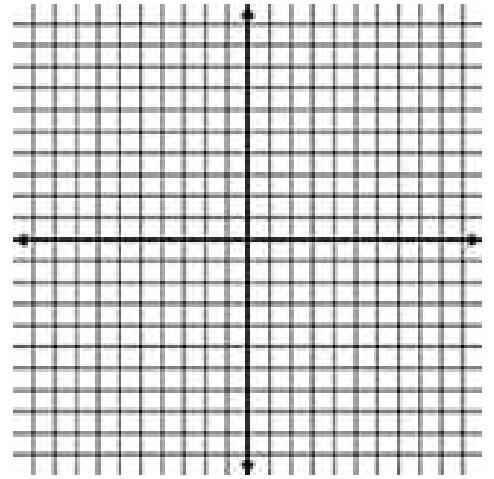
c. For what value of x is $f(x) = 3$?

d. The domain of f

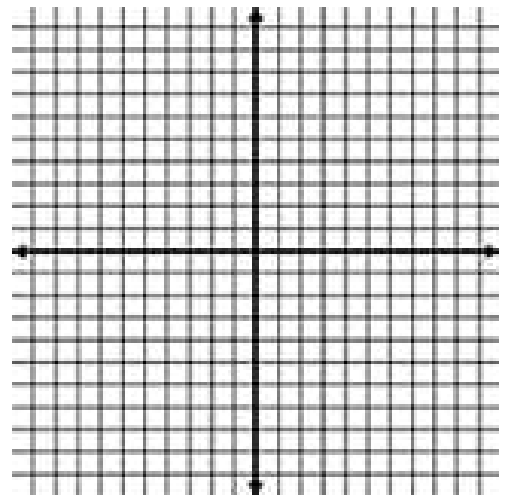
e. The range of f

Example 3: Graph the following functions by plotting points and identify the domain and range.

a. $f(x) = -x - 2$



b. $H(x) = x^2 + 1$



Example 4: Consider the function $f(x) = \frac{x^2 + 2}{x + 4}$.

a. Is the point $\left(1, \frac{3}{5}\right)$ on the graph?

b. If $x = 0$, what is $f(x)$? What point is on the graph of f ?

c. If $f(x) = \frac{1}{2}$, what is x ? What point(s) are on the graph of f ?

d. What is the domain of f ?

e. List the x -intercepts, if any, of the graph of f .

f. List the y -intercepts, if any, of the graph of f .

2.3: PROPERTIES OF FUNCTIONS

When you are done with your homework you should be able to...

- π Determine Even and Odd functions from a Graph
- π Identify Even and Odd functions from the Equation
- π Use a Graph to Determine Where a Function is Increasing, Decreasing, or Constant
- π Use a Graph to Locate Local Maxima and Local Minima
- π Use a Graph to Locate the Absolute Maximum and Absolute Minimum
- π Use a Graphing Utility to Approximate Local Maxima and Local Minima
- π Find the Average Rate of Change of a Function

WARM-UP: Test the equation $y = -x^2 + 3$ for symmetry with respect to the x -axis, y -axis, and the origin.

EVEN FUNCTIONS

A function f is _____ if, for every number _____ in its domain, the number _____ is also in the domain and

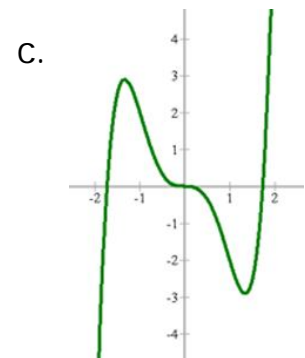
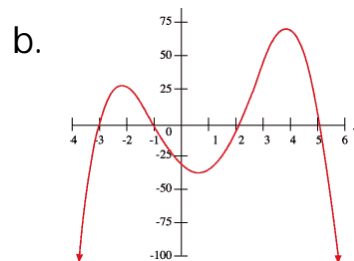
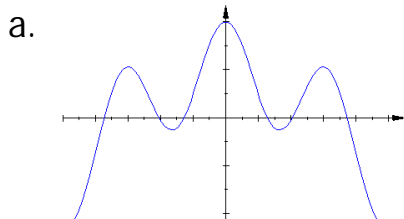
ODD FUNCTIONS

A function f is _____ if, for every number _____ in its domain, the number _____ is also in the domain and

THEOREM

A function is _____ if and only if its graph is symmetric with respect to the _____. A function is _____ if and only if its graph is symmetric with respect to the _____.

Example 1: Determine whether each graph given below is the graph of an even function, an odd function, or a function that is neither even nor odd.



Example 2: Determine algebraically whether each function is even, odd, or neither.

a. $h(x) = 3x^3 + 5$

b. $F(x) = \frac{2x}{|x|}$

c. $f(x) = 2x^4 - x^2$

INCREASING/DECREASING/CONSTANT INTERVALS OF A FUNCTION

A function f is _____ on an open _____ if, for any choice of _____ and _____ in I , with _____, we have _____.

A function f is _____ on an open _____ if, for any choice of _____ and _____ in I , with _____, we have _____.

A function f is _____ on an open _____ if, for all choices of _____ in I , the values of _____ are _____.

**NOTE: We describe the behavior of a graph in terms of its _____!!!

LOCAL EXTREMA

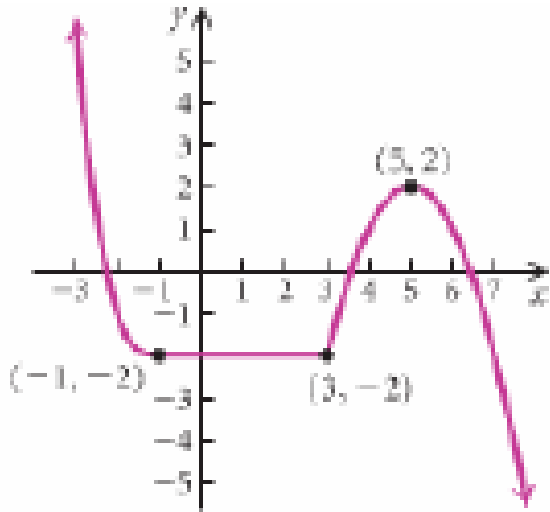
A function f has a _____ at _____ if there is an open interval I containing c so that for all x in I , _____. We call _____ a _____ of _____.

A function f has a _____ at _____ if there is an open interval I containing c so that for all x in I , _____. We call _____ a _____ of _____.

**NOTE: The word _____ is used to suggest that it is only near _____, that is, in some open interval containing c , that the value of _____ has these properties.

**NOTE: The _____ is the local maximum or minimum value and it *occurs* at some _____.

Example 3: Consider the graph of the function given below.



- On what interval(s) is f increasing?
- On what interval(s) is f decreasing?
- On what interval(s) is f constant?
- List the local minima.
- List the ordered pair(s) where a local minimum occurs.
- List the local maxima.
- List the ordered pair(s) where a local maximum occurs.

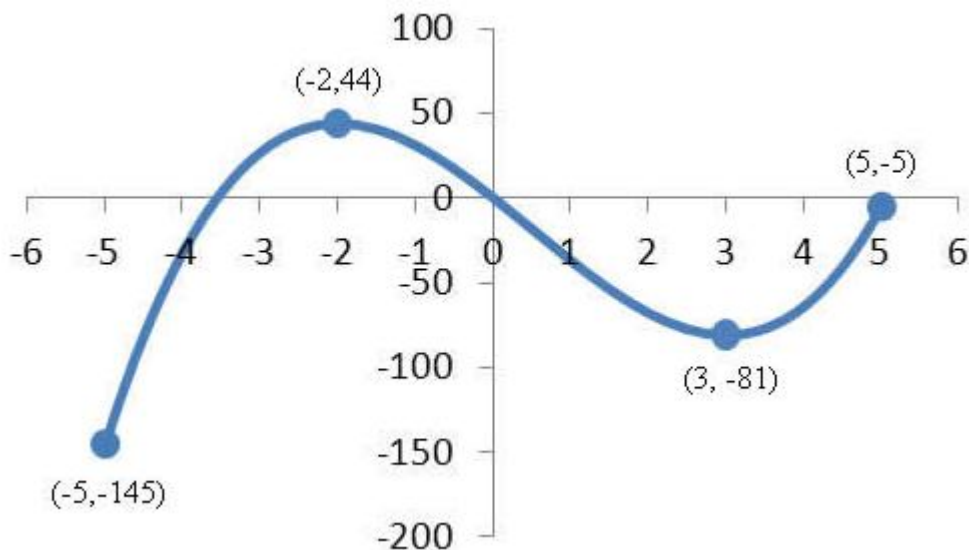
ABSOLUTE EXTREMA

Let f denote a function defined on some interval I . If there is a number _____ in I for which _____ for all x in I , then _____ is the _____ of _____ on _____ and we say the _____ of _____ occurs at _____.

If there is a number _____ in I for which _____ for all x in I , then _____ is the _____ of _____ on _____ and we say the _____ of _____ occurs at _____.

Example 4: Find the absolute minimum and the absolute maximum, if they exist, of the following graphs below.

a.



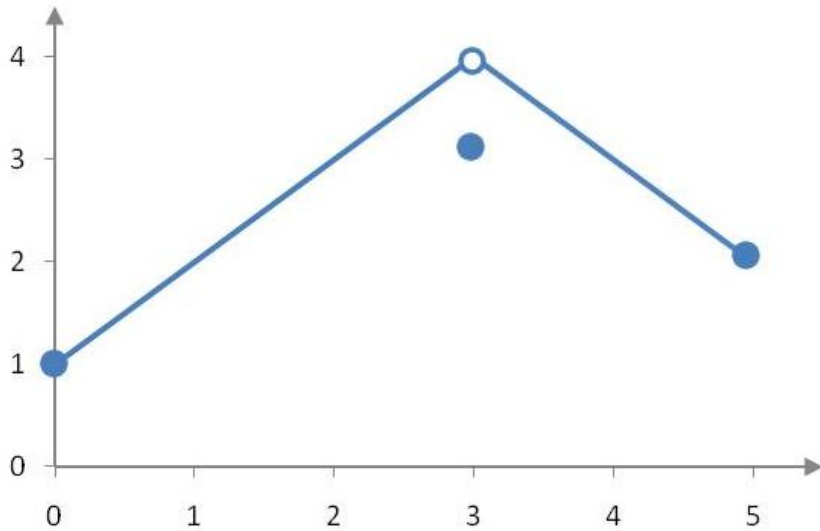
The absolute minimum is _____.

The absolute minimum occurs at _____.

The absolute maximum is _____.

The absolute maximum occurs at _____.

b.



The absolute minimum is _____.

The absolute minimum occurs at _____.

The absolute maximum is _____.

The absolute maximum occurs at _____.

EXTREME VALUE THEOREM

If f is a continuous function whose domain is a closed interval $[a, b]$, then f has an _____ and an _____ on $[a, b]$.

**NOTE: You can consider a continuous function to be a function whose graph has no _____ or _____ and can be _____ without lifting the pencil from the paper.

AVERAGE RATE OF CHANGE

If _____ and _____, _____, are in the domain of a function $y = f(x)$, the

_____ of _____ from _____ to

_____ is defined as

Average rate of change =

Example 5: Find the average rate of change of $f(x) = -x^3 + 1$

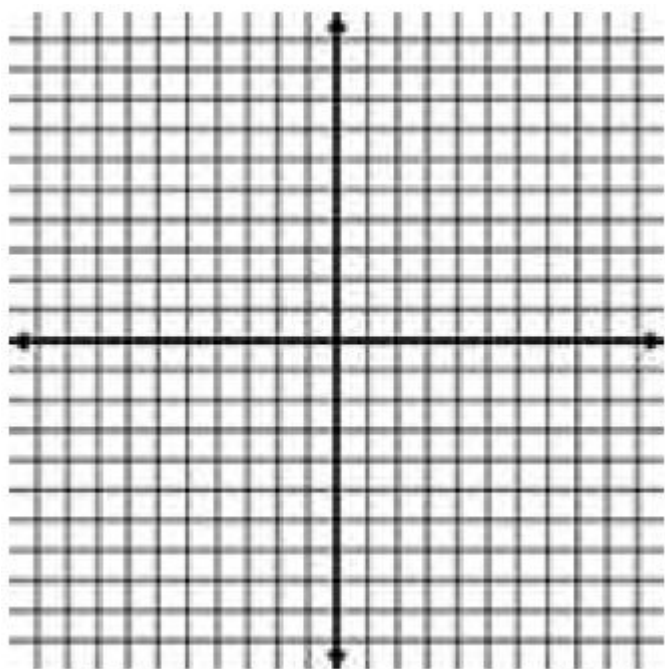
a. From 0 to 2

b. From 1 to 3

c. From -1 to 1

THEOREM: SLOPE OF THE SECANT LINE

The _____ of _____ of a function from _____ to _____ equals the _____ of the _____ line containing the two points _____ and _____ on its graph.



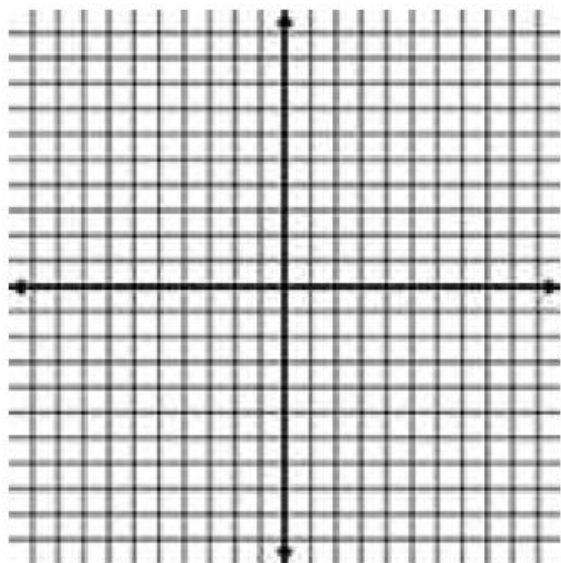
Example 6: Consider $h(x) = -2x^2 + x$

Find an equation of the secant line containing the x-coordinates 0 and 3.

THE LIBRARY OF FUNCTIONS

Example 1: Consider the function $f(x) = b$.

- Determine whether $f(x) = b$ is even, odd, or neither. State whether the graph is symmetric with respect to the y -axis or symmetric with respect to the origin.
- Determine the intercepts, if any, of the graph of $f(x) = b$.
- Graph $f(x) = b$ by hand.



PROPERTIES OF $f(x) = b$

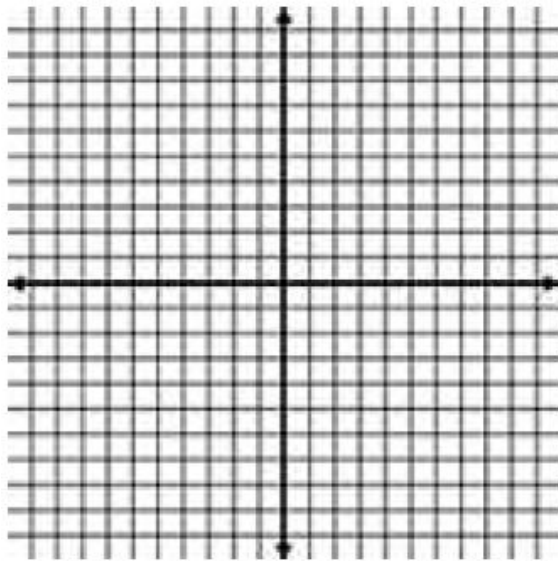
- The domain is the set of _____ numbers. The range of f is the set consisting of a single number _____.
- The y -intercept of the graph of $f(x) = b$ is _____.
- The graph is a _____ line. The function is _____ with respect to the _____. The function is _____.

Example 2: Consider the function $f(x) = x$.

a. Determine whether $f(x) = x$ is even, odd, or neither. State whether the graph is symmetric with respect to the y -axis or symmetric with respect to the origin.

b. Determine the intercepts, if any, of the graph of $f(x) = x$.

c. Graph $f(x) = x$ by hand.

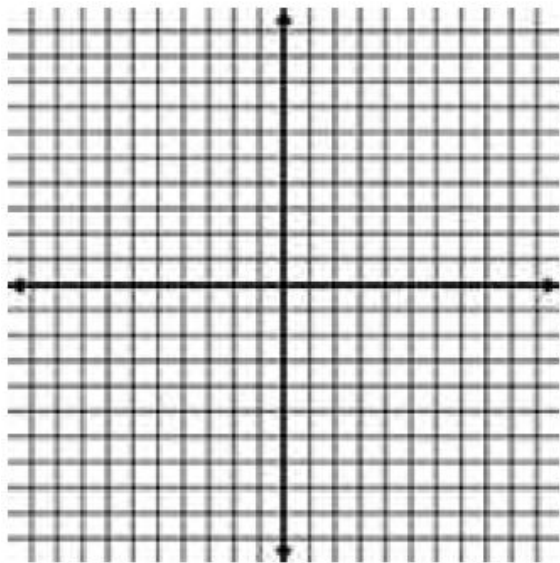


PROPERTIES OF $f(x) = x$

1. The domain and range are the set of _____ numbers.
2. The x -intercept of the graph of $f(x) = x$ is _____. The y -intercept of the graph of $f(x) = x$ is _____.
3. The graph is _____ with respect to the _____.
4. The function is _____.
5. The function is _____ on the interval _____.

Example 3: Consider the function $f(x) = x^2$.

- Determine whether $f(x) = x^2$ is even, odd, or neither. State whether the graph is symmetric with respect to the y -axis or symmetric with respect to the origin.
- Determine the intercepts, if any, of the graph of $f(x) = x^2$.
- Graph $f(x) = x^2$ by hand.



PROPERTIES OF $f(x) = x^2$

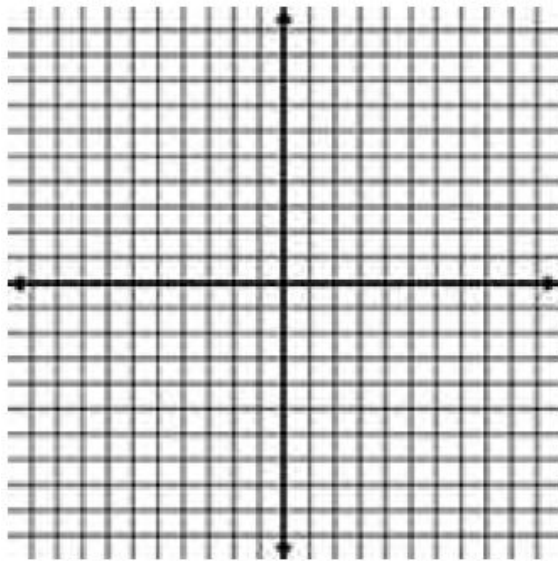
- The domain is the set of _____ numbers. The range is the set of _____ real numbers.
- The x -intercept of the graph of $f(x) = x^2$ is _____. The y -intercept of the graph of $f(x) = x^2$ is _____.
- The graph is _____ with respect to the _____.
- The function is _____.
- The function is _____ on the interval _____ and _____ on the interval _____.

Example 4: Consider the function $f(x) = x^3$.

a. Determine whether $f(x) = x^3$ is even, odd, or neither. State whether the graph is symmetric with respect to the y -axis or symmetric with respect to the origin.

b. Determine the intercepts, if any, of the graph of $f(x) = x^3$.

c. Graph $f(x) = x^3$ by hand.



PROPERTIES OF $f(x) = x^3$

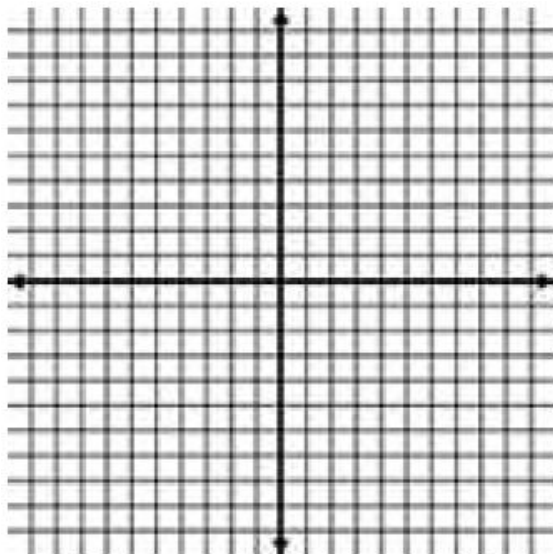
1. The domain and range are the set of _____ numbers.
2. The x -intercept of the graph of $f(x) = x^3$ is _____. The y -intercept of the graph of $f(x) = x^3$ is _____.
3. The graph is _____ with respect to the _____.
4. The function is _____.
5. The function is _____ on the interval _____.

Example 5: Consider the function $f(x) = \sqrt{x}$.

a. Determine whether $f(x) = \sqrt{x}$ is even, odd, or neither. State whether the graph is symmetric with respect to the y -axis or symmetric with respect to the origin.

b. Determine the intercepts, if any, of the graph of $f(x) = \sqrt{x}$.

c. Graph $f(x) = \sqrt{x}$ by hand.



PROPERTIES OF $f(x) = \sqrt{x}$

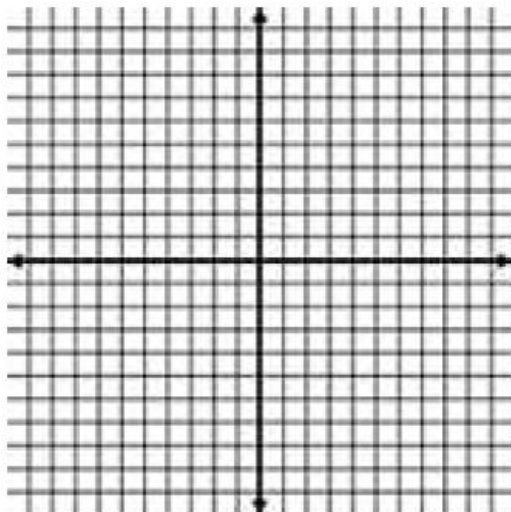
1. The domain and range are the set of _____
_____ numbers.
2. The x -intercept of the graph of $f(x) = \sqrt{x}$ is _____. The y -intercept of the graph of $f(x) = \sqrt{x}$ is _____.
3. The function is _____ nor _____.
4. The function is _____ on the interval _____.
5. The function has an _____ of _____ at _____.

Example 6: Consider the function $f(x) = \sqrt[3]{x}$.

a. Determine whether $f(x) = \sqrt[3]{x}$ is even, odd, or neither. State whether the graph is symmetric with respect to the y-axis or symmetric with respect to the origin.

b. Determine the intercepts, if any, of the graph of $f(x) = \sqrt[3]{x}$.

c. Graph $f(x) = \sqrt[3]{x}$ by hand.



PROPERTIES OF $f(x) = \sqrt[3]{x}$

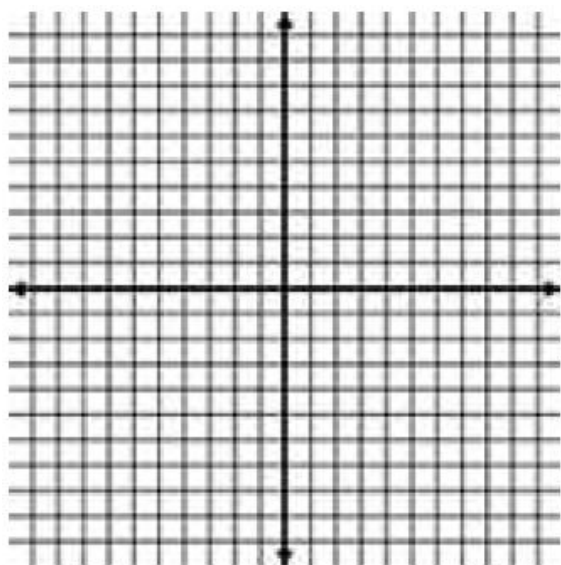
1. The domain and range are the set of _____ numbers.
2. The x-intercept of the graph of $f(x) = \sqrt[3]{x}$ is _____. The y-intercept of the graph of $f(x) = \sqrt[3]{x}$ is _____.
3. The graph is _____ with respect to the _____. The function is _____.
4. The function is _____ on the interval _____.
5. The function does not have any local _____ or local _____.

Example 7: Consider the function $f(x) = \frac{1}{x}$.

a. Determine whether $f(x) = \frac{1}{x}$ is even, odd, or neither. State whether the graph is symmetric with respect to the y -axis or symmetric with respect to the origin.

b. Determine the intercepts, if any, of the graph of $f(x) = \frac{1}{x}$.

c. Graph $f(x) = \frac{1}{x}$ by hand.



PROPERTIES OF $f(x) = \frac{1}{x}$

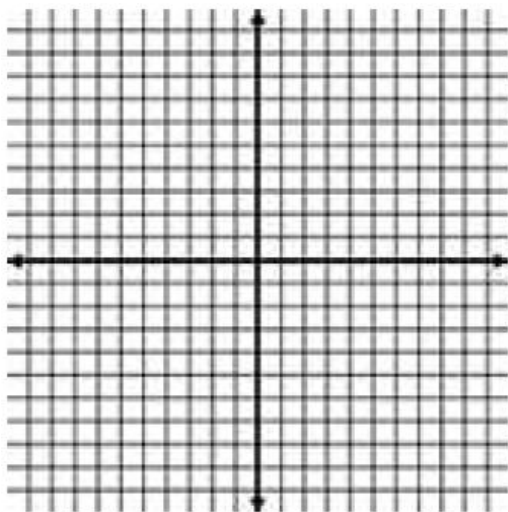
1. The domain and range are the set of all _____ real numbers.
2. The graph of $f(x) = \frac{1}{x}$ has _____ intercepts.
3. The graph is _____ with respect to the _____.
4. The function is _____.
5. The function is _____ on the interval _____ and _____ on the interval _____.

Example 8: Consider the function $f(x) = |x|$.

a. Determine whether $f(x) = |x|$ is even, odd, or neither. State whether the graph is symmetric with respect to the y -axis or symmetric with respect to the origin.

b. Determine the intercepts, if any, of the graph of $f(x) = |x|$.

c. Graph $f(x) = |x|$ by hand.



PROPERTIES OF $f(x) = |x|$

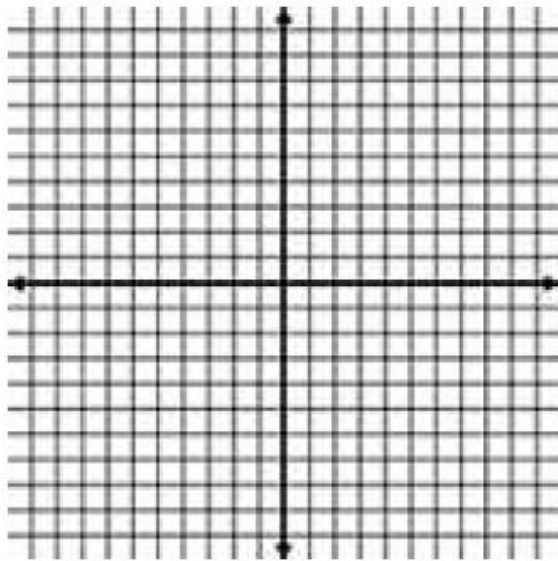
1. The domain is the set of _____ numbers. The range of f is _____.
2. The x -intercept of the graph of $f(x) = |x|$ is _____. The y -intercept of the graph of $f(x) = |x|$ is _____.
3. The graph is _____ with respect to the _____. The function is _____.
4. The function is _____ on the interval _____ and _____ on the interval _____.
5. The function has an _____ of _____ at _____.

Example 9: Consider the function $f(x) = \text{int}(x)$.

- a. Determine whether $f(x) = \text{int}(x)$ is even, odd, or neither. State whether the graph is symmetric with respect to the y -axis or symmetric with respect to the origin.

- b. Determine the intercepts, if any, of the graph of $f(x) = \text{int}(x)$.

- c. Graph $f(x) = \text{int}(x)$ by hand.

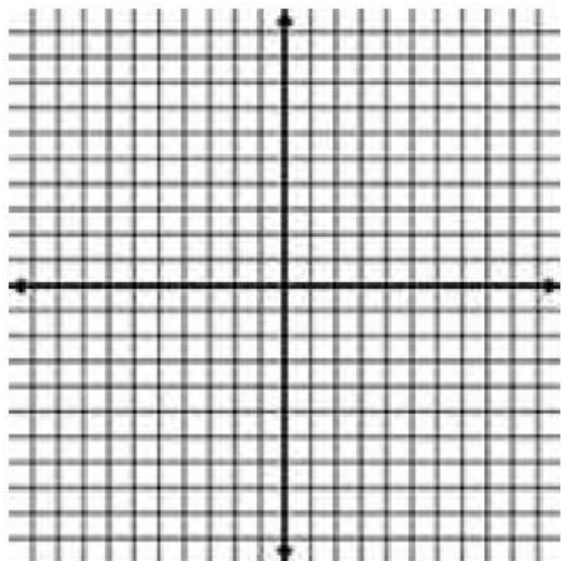


PROPERTIES OF $f(x) = \text{int}(x)$

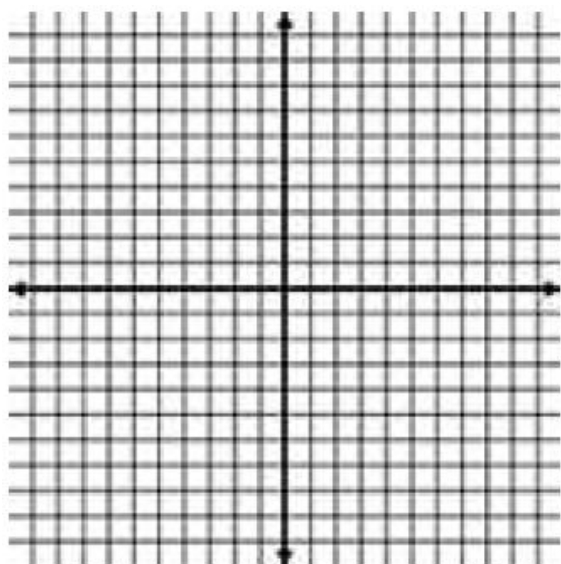
1. The domain is the set of all _____ numbers. The range is the set of _____.
2. The x -intercepts lie on the interval _____. The y -intercept is _____.
3. The function is _____ nor _____.
4. The function is _____ on every interval of the form _____, for _____ an _____.

Example 10: Sketch the graph of the following functions. Find the domain of each function. Locate any intercepts. Based on the graph, find the range. Is f continuous on its domain?

a.
$$f(x) = \begin{cases} -3x & \text{if } x < -1 \\ 0 & \text{if } x = 1 \\ 2x^2 + 1 & \text{if } x > 1 \end{cases}$$



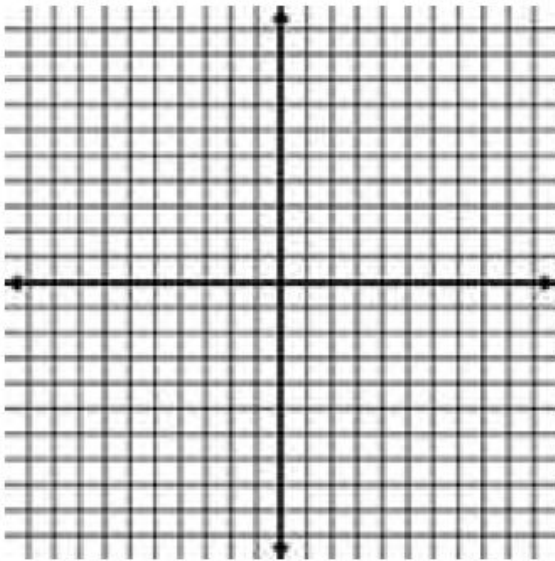
b.
$$f(x) = \begin{cases} 2-x & \text{if } -3 \leq x < 1 \\ \sqrt{x} & \text{if } x > 1 \end{cases}$$



APPLICATION

The short-term (no more than 24 hours) parking fee F (in dollars) for parking x hours at O'Hare International Airport's main parking garage can be modeled by the function

$$F(x) = \begin{cases} 2 & \text{if } 0 < x \leq 1 \\ 4 & \text{if } 1 < x \leq 3 \\ 10 & \text{if } 3 < x \leq 4 \\ 5 \operatorname{int}(x+1) + 2 & \text{if } 4 < x < 9 \\ 51 & \text{if } 9 \leq x \leq 24 \end{cases}$$



Determine the fee for parking in the short-term parking garage for

- 2 hours
- 7 hours
- 15 hours
- 8 hours and 24 minutes

2.5: GRAPHING TECHNIQUES: TRANSFORMATIONS

When you are done with your homework, you should be able to...

- π Graph Functions Using Vertical and Horizontal Shifts
- π Graph Functions Using Compressions and Stretches
- π Graph Functions Using Reflections about the x-axis or y-axis

WARM-UP:

1. Consider the functions

$$Y_1 = x^3$$

$$Y_2 = x^3 + 4$$

$$Y_3 = x^3 - 4$$

- a. Graph each of the following functions on the same screen.

- b. Create a table of values for Y_1 , Y_2 , and Y_3 .

- c. Describe Y_2 in terms of Y_1 .

- d. Describe Y_3 in terms of Y_1 .

2. Consider the functions

$$Y_1 = x^3$$

$$Y_2 = (x - 4)^3$$

$$Y_3 = (x + 4)^3$$

a. Graph each of the following functions on the same screen.

b. Create a table of values for Y_1 , Y_2 , and Y_3 .

c. Describe Y_2 in terms of Y_1 .

d. Describe Y_3 in terms of Y_1 .

3. Consider the functions

$$Y_1 = x^4$$

$$Y_2 = 2x^4$$

$$Y_3 = \frac{1}{2}x^4$$

a. Graph each of the following functions on the same screen.

b. Create a table of values for Y_1 , Y_2 , and Y_3 .

c. Describe Y_2 in terms of Y_1 .

d. Describe Y_3 in terms of Y_1 .

4. Consider the functions

$$Y_1 = x^4$$

$$Y_2 = -x^4$$

a. Graph each of the following functions on the same screen.

b. Create a table of values for Y_1 and Y_2 .

c. Describe Y_2 in terms of Y_1 .

5. Consider the functions

$$Y_1 = \sqrt{x}$$

$$Y_2 = \sqrt{-x}$$

a. Graph each of the following functions on the same screen.

b. Create a table of values for Y_1 and Y_2 .

c. Describe Y_2 in terms of Y_1 .

SUMMARY OF GRAPHING TECHNIQUES

TO GRAPH:	DRAW THE GRAPH OF f AND:	FUNCTIONAL CHANGE TO $f(x)$
-----------	----------------------------	-----------------------------

VERTICAL SHIFTS

$y = f(x) + k, k > 0$	_____ the graph of f by _____ units.	_____ k to $f(x)$.
$y = f(x) - k, k > 0$	_____ the graph of f by _____ units.	_____ k from $f(x)$.

HORIZONTAL SHIFTS

$y = f(x + h), h > 0$	_____ the graph of f to the _____ units.	_____ x by _____.
$y = f(x - h), h > 0$	_____ the graph of f to the _____ units.	_____ x by _____.

COMPRESSING OR STRETCHING

$y = af(x), a > 0$	_____ each _____ of $y = f(x)$ by _____.	_____ $f(x)$ by _____.
	_____ the graph of f _____ if $a > 1$.	
	_____ the graph of f _____ if $0 < a < 1$.	

$y = f(ax), a > 0$	<p>_____ each _____ of $y = f(x)$ by _____.</p> <p>_____ the graph of f _____ if $0 < a < 1$.</p> <p>_____ the graph of f _____ if $a > 1$.</p>	<p>_____ x by _____.</p>
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REFLECTION ABOUT THE x -AXIS

$y = -f(x)$	<p>_____ the graph of f about the _____.</p>	<p>_____ $f(x)$ by _____.</p>
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REFLECTION ABOUT THE y -AXIS

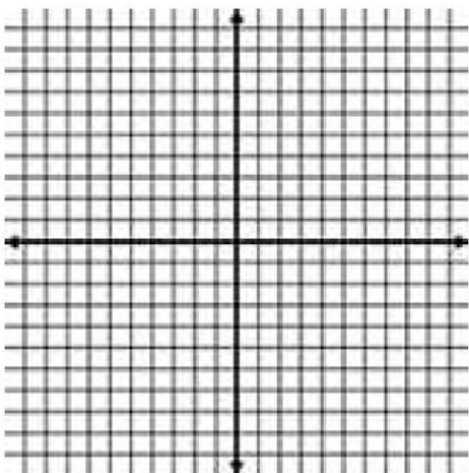
$y = f(-x)$	<p>_____ the graph of f about the _____.</p>	<p>_____ x by _____.</p>
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Example 1: Write the function whose graph is $y = x^2$, but is

- a. Shifted to the left 8 units.
- b. Shifted down 8 units.
- c. Reflected about the x -axis.
- d. Shifted to the up 8 units.
- e. Vertically compressed by a factor of 8.
- f. Horizontally stretched by a factor of 8 units.
- g. Shifted to the right 8 units.
- e. Reflected about the y -axis.

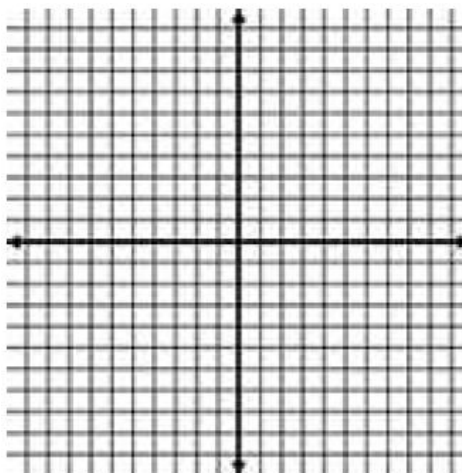
Example 2: Graph each function using the techniques of shifting, compressing, stretching, and/or reflecting. Start with the graph of the basic function and show all stages. Be sure to show at least three key points. Find the domain and range of each function.

a. $h(x) = \sqrt{x+1}$



Domain: _____

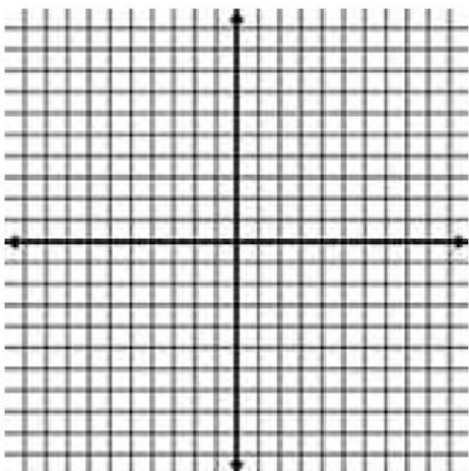
Range: _____



Domain: _____

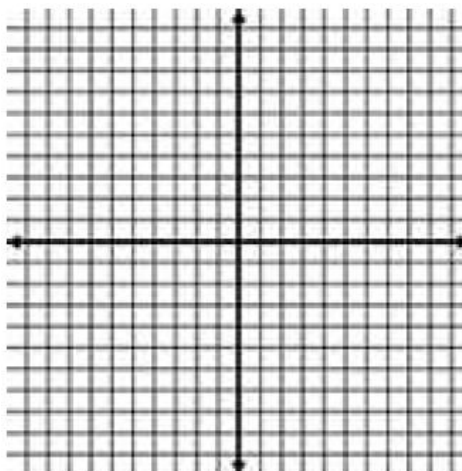
Range: _____

b. $f(x) = \frac{1}{2}\sqrt{x}$



Domain: _____

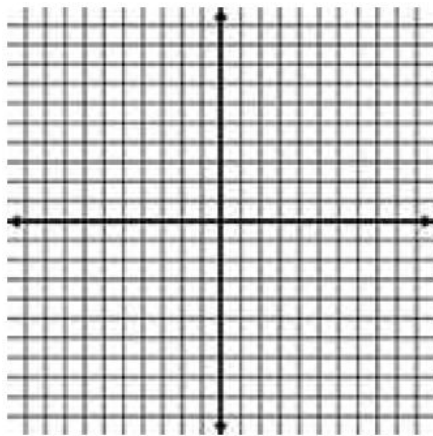
Range: _____



Domain: _____

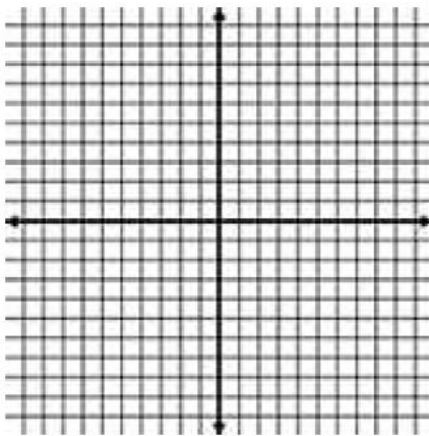
Range: _____

c. $g(x) = \sqrt{-x} - 2$



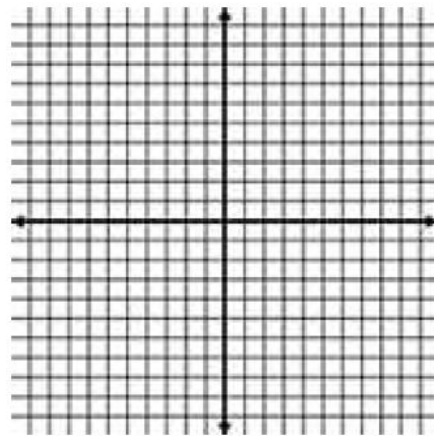
Domain: _____

Range: _____



Domain: _____

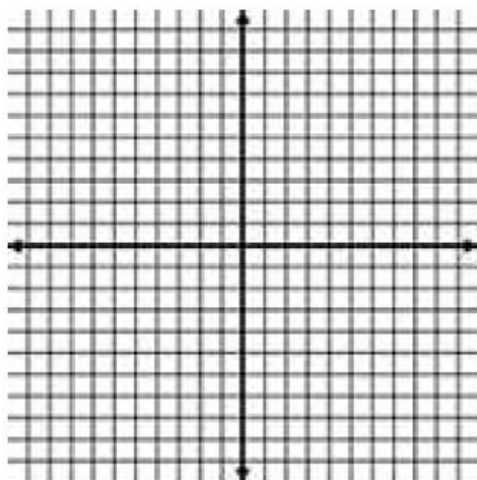
Range: _____



Domain: _____

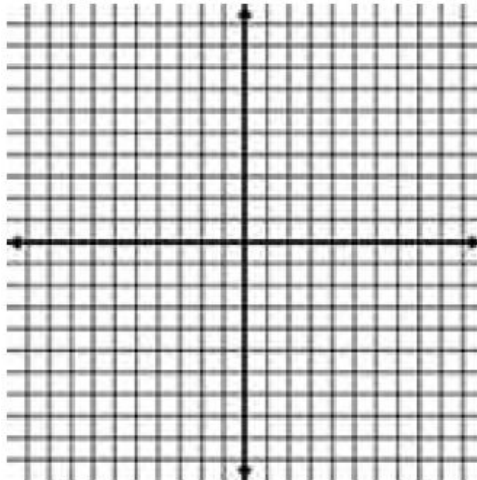
Range: _____

d. $h(x) = \text{int}(-x)$



Domain: _____

Range: _____



Domain: _____

Range: _____

Example 3: Suppose that the function $y = f(x)$ is decreasing on the interval $(-2, 7)$.

a. Over what interval is the graph of $y = f(x+2)$ decreasing?

b. Over what interval is the graph of $y = f(x-5)$ decreasing?

c. What can be said about the graph of $y = -f(x)$?

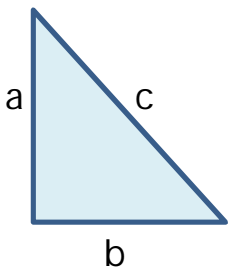
2.6: MATHEMATICAL MODELS: BUILDING FUNCTIONS

When you are done with your homework you should be able to...

π Build and Analyze Functions

WARM-UP: Complete the following statements.

1. The sum of angles in a triangle is _____.
2. The distance between the ordered pairs (x_1, y_1) and (x_2, y_2) is _____.
3. Distance = _____.
4. The area of a rectangle is _____.
5. Perimeter is the _____ of the _____ of the _____ of a polygon.
6. The area of a circle is _____.
7. The Pythagorean Theorem states: _____.



8. The volume of a right circular cylinder is _____.
9. The volume of a right circular cone is _____.
10. The volume of a sphere is _____.
11. The volume of a right rectangular prism is _____.
12. The volume of a right rectangular pyramid is _____.

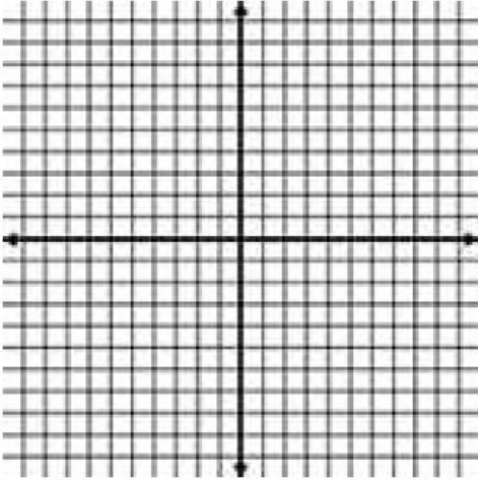
Example 1: Let $P = (x, y)$ be a point on the graph of $y = \frac{1}{x}$.

a. Express the distance d from P to the origin as a function of x .

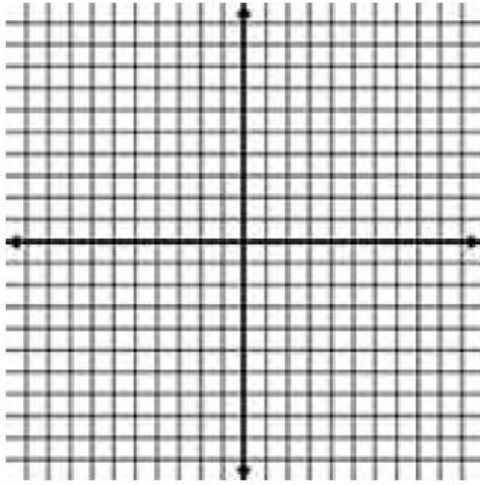
b. Use a graphing utility to graph $d = d(x)$.

c. For what values of x is d smallest?

Example 2: A right triangle has one vertex on the graph of $y = 9 - x^2$, $x > 0$, at (x, y) , another at the origin, and the third on the positive x -axis at $(x, 0)$. Express the area A of the triangle as a function of x .

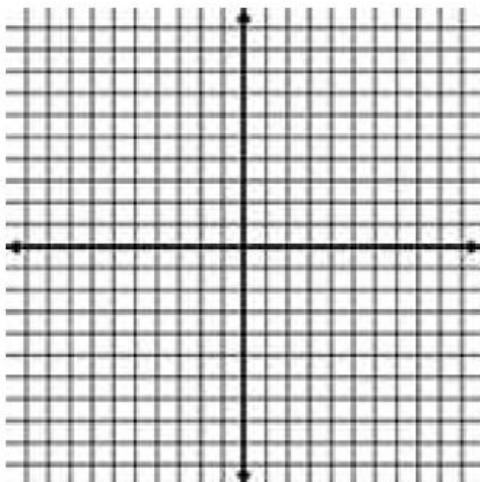


Example 3: A rectangle is inscribed in a semicircle of radius 2. Let $P = (x, y)$ be the point in quadrant I that is a vertex of the rectangle and is on the circle.

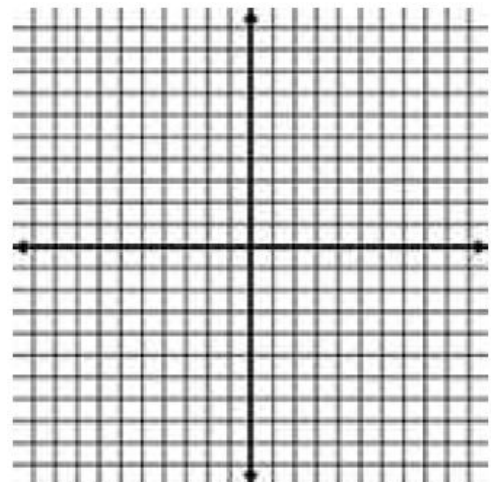


- Express the area A of the rectangle as a function of x .
- Express the perimeter p of the rectangle as a function of x .

c. Graph $A = A(x)$. For what value of x is A largest?



d. Graph $p = p(x)$. For what value of x is p largest?



Example 3: Two cars leave an intersection at the same time. One is headed south at a constant speed of 30 mph and the other is headed west at a constant speed of 40 mph. Build a model that expresses the distance d between the cars as a function of time t .

3.1: LINEAR FUNCTIONS AND THEIR PROPERTIES

When you are done with your homework you should be able to...

- π Graph Linear Functions
- π Use Average Rate of Change to Identify Linear functions
- π Determine Whether a Linear Function is Increasing, Decreasing, or Constant
- π Build Linear Models From Verbal Descriptions

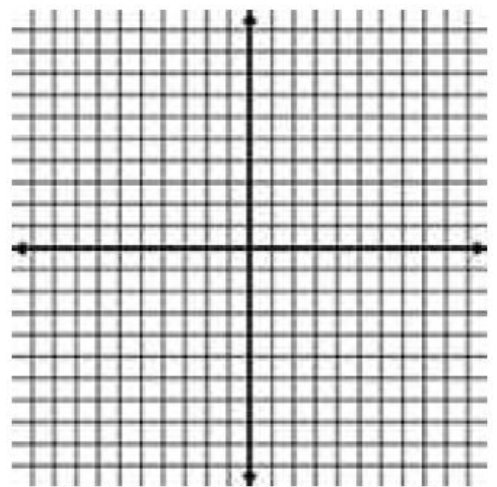
WARM-UP: Write the equation of the line which passes through the points $(-3,2)$ and $(5,7)$.

LINEAR FUNCTION

A linear function is a function of the form

The graph of a linear function is a _____ with slope _____ and y-intercept _____. Its domain is the set of all _____ numbers.

Example 1: Graph the linear function: $f(x) = -\frac{2}{3}x - 4$



AVERAGE RATE OF CHANGE OF A LINEAR FUNCTION

Linear functions have a _____ average rate of change. The average rate of change of _____ is

PROOF:

Example 2: Determine whether the given function is linear or nonlinear. If it is linear, determine the equation of the line.

a.

x	$y = f(x)$
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4

b.

x	$y = f(x)$
-4	8
-2	4
0	0
2	-4
4	-8

INCREASING, DECREASING, AND CONSTANT LINEAR FUNCTIONS

A linear function _____ is...

_____ over its domain if its _____, _____, is _____.

_____ over its domain if its _____, _____, is _____.

_____ over its domain if its _____, _____, is _____.

Example 3: Determine whether the following linear functions are increasing, decreasing, or constant.

a. $f(x) = 2 - 4x$

b. $h(z) = -6$

c. $g(t) = 0.02t - 0.35$

Example 4: Consider the following linear function.

$y = g(x)$

a. Solve $g(x) = -1$

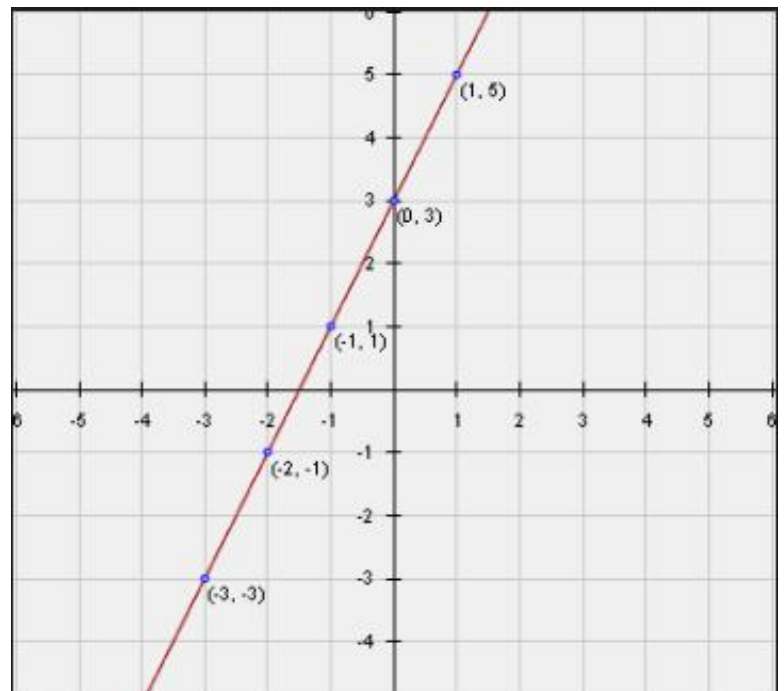
b. Solve $g(x) = 0$

c. Solve $g(x) \leq 3$

d. Solve $g(x) = 5$

e. Solve $g(x) > -1$

f. Solve $0 < g(x) < 5$



2. Suppose that the quantity supplied S and quantity demanded D of hot dogs at a baseball game are given by the following functions:

$$S(p) = -2000 + 3000p$$

$$D(p) = 10,000 - 1000p$$

where p is the price of a hot dog.

- Find the equilibrium price for hot dogs at the baseball game. What is the equilibrium quantity?
- Determine the prices for which quantity demanded is less than quantity supplied.
- What do you think will eventually happen to the price of hot dogs if quantity demanded is less than quantity supplied?

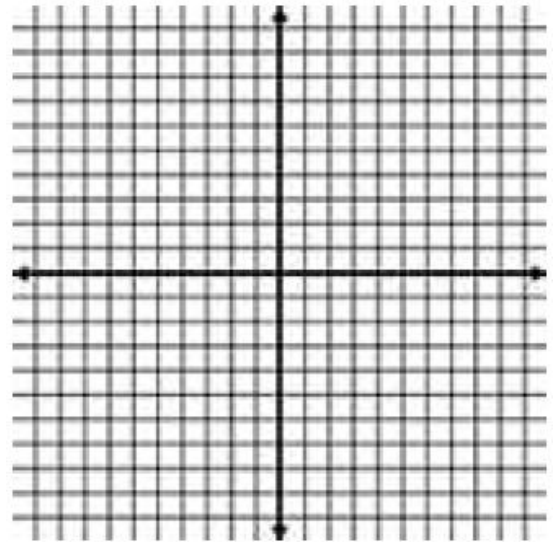
3.3: QUADRATIC FUNCTIONS AND THEIR PROPERTIES

When you are done with your homework, you should be able to...

- π Graph a Quadratic Function Using Transformations
- π Identify the Vertex and Axis of Symmetry of a Quadratic Function
- π Graph a Quadratic Function Using Its Vertex, Axis, and Intercepts
- π Find a Quadratic Function Given Its Vertex and One Other Point
- π Find the Maximum or Minimum Value of a Quadratic Function

WARM-UP:

1. Graph $f(x) = x^2$.



2. Complete the square of the expression $x^2 + 6x - 1$

Graphs like the one we just did in the warm-up problem are the graphs of _____ functions, commonly called _____. Parabolas open upward if the coefficient to the squared term is _____ and downward if the coefficient to the squared term is _____. Parabolas have a “fold” line, that is, they have _____ symmetry about a vertical line. This vertical line is found when you find the ordered pair where the _____ or _____ is located. This ordered pair is called the _____ of the quadratic function.

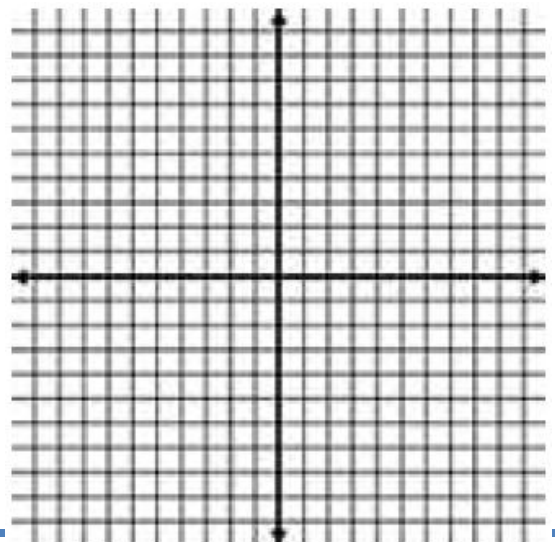
QUADRATIC FUNCTION

A quadratic function is a function of the form

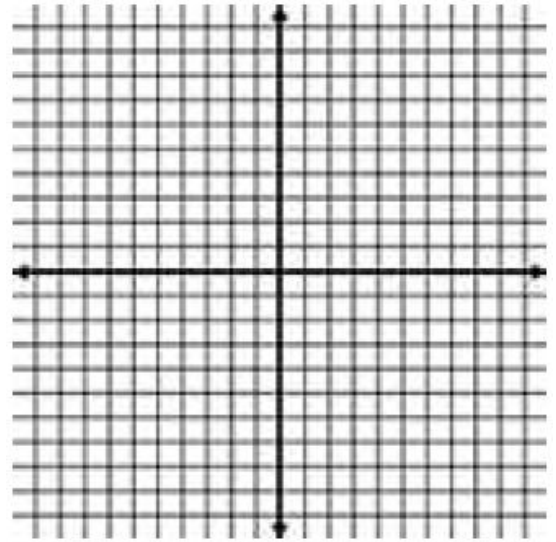
where a , b , and c are real numbers and _____. The domain of a quadratic function is the set of _____ numbers.

Example 1: Graph using transformations.

a. $f(x) = 2x^2 + 4$.



b. $f(x) = -2x^2 + 6x + 2$



Now consider any quadratic function $f(x) = ax^2 + bx + c$.

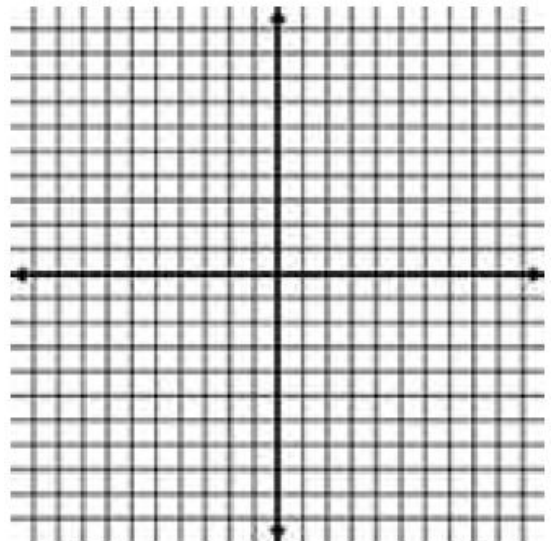
Based on these results, we conclude...

If _____, and _____, then

where the vertex is the ordered pair _____. If _____, the _____ occurs at the vertex and if _____ the _____ occurs at the vertex.

Example 2: Consider the function $f(x) = -(x-3)^2 + 6$.

- What is the vertex?
- What is the axis of symmetry?
- Find the x -intercept(s).
- Find the y -intercept.
- Sketch the graph.



Oftentimes, we are given quadratic equations in the form _____.

When this happens, it is easier to use the fact the _____ and find

_____ by evaluating _____.

PROPERTIES OF THE GRAPH OF A QUADRATIC FUNCTION

$$f(x) = ax^2 + bx + c$$

Vertex: _____

Axis of Symmetry: _____

If the parabola opens upward, _____ and the vertex is a
_____ point.

If the parabola opens downward, _____ and the vertex is a
_____ point.

Example 3: Find the coordinates of the vertex for the parabola defined by the given quadratic function.

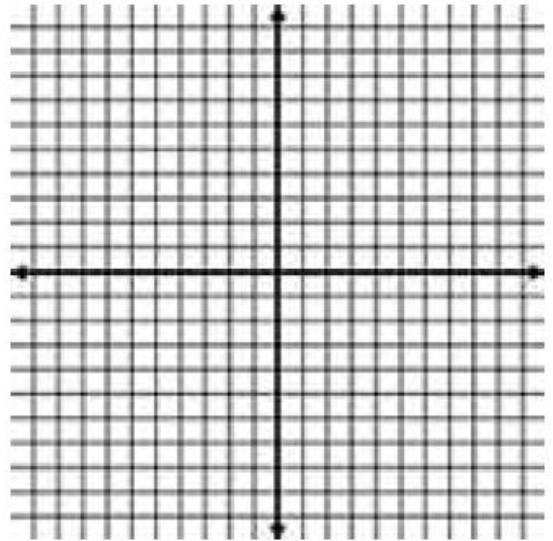
a. $f(x) = 3x^2 - 12x + 1$

b. $f(x) = -2x^2 + 7x - 4$

c. $f(x) = -3(x - 2)^2 + 12$

Example 4: Consider the function $f(x) = 3x^2 - 8x + 2$.

a. What is the vertex?



b. What is the axis of symmetry?

c. Find the x-intercept(s).

d. Find the y-intercept.

e. Sketch the graph.

Example 5: The graph of the function $f(x) = ax^2 + bx + c$ has vertex at $(1, 4)$ and passes through the point $(-1, -8)$. Find a , b , and c .

STEPS FOR GRAPHING A QUADRATIC FUNCTION $f(x) = ax^2 + bx + c$, $a \neq 0$

Option 1

1. Complete the square in x to write the equation in the form _____.
2. Graph the function in stages using _____.

Option 2

1. Determine whether the parabola opens up (_____) or down (_____).
2. Determine the vertex: _____
3. Determine the axis of symmetry _____.
4. Find the _____, if any.
 - a. If _____, the graph of the quadratic function has _____.
 - b. If _____, the _____ is the _____.
 - c. If _____, there are _____.
5. Determine an additional point using _____.
6. Plot the points and sketch the graph.

APPLICATIONS

1. Find the point on the line $y = x + 1$ that is closest to point $(4, 1)$.

2. The John Deere Company has found that the revenue, in dollars, from sales of riding mowers is a function of the unit price p , in dollars, that it charges.

If the revenue R is $R(p) = -\frac{1}{2}p^2 + 1900p$ what unit price p should be charged to maximize revenue? What is the maximum revenue?

3.4: BUILD QUADRATIC MODELS FROM VERBAL DESCRIPTIONS

When you are done with your homework, you should be able to...

π Build Quadratic Models From Verbal Descriptions

WARM-UP: Find the vertex of the quadratic function $f(x) = -2x^2 - x + 5$.

Example 1: The price p (in dollars) and the quantity x sold of a certain product obey the demand equation $p = -\frac{1}{3}x + 100$.

- Find a model that expresses the revenue R as a function of x .
- What is the domain of R ?
- What is the revenue if 100 units are sold?
- What quantity x maximizes revenue? What is the maximum revenue?
- What price should the company charge to maximize revenue?

Example 2: A farmer with 2000 meters of fencing wants to enclose a rectangular plot that borders on a straight highway. If the farmer does not fence the side along the highway, what is the largest area that can be enclosed?

Example 3: A parabolic arch has a span of 120 feet and a maximum height of 25 feet. Choose suitable rectangular axes and find an equation of the parabola. Then calculate the height of the arch at points 10 feet, 20 feet, and 40 feet from the center.

d. Using a graphing calculator, graph the function h , $0 \leq x \leq 350$.

e. Use a graphing calculator to verify the results obtained in parts b and c.

f. When the height of the projectile is 50 feet above the ground, how far has it traveled horizontally?

3.5: INEQUALITIES INVOLVING QUADRATIC FUNCTIONS

When you are done with your homework, you should be able to...

π Solve Inequalities Involving a Quadratic Function

WARM-UP: Find the zeroes of $f(x) = 3x^2 - x - 5$.

STEPS TO SOLVE A QUADRATIC INEQUALITY

1. Find the _____ of the quadratic function _____.
2. Draw a number line, using the _____ to separate the number line into intervals.
3. Choose a number from each interval and evaluate the number in _____.
 - a. If you get a positive result, that interval is the solution for inequalities with _____ or _____.
 - b. If you get a negative result, that interval is the solution for inequalities with _____ or _____.
4. Write your result in set and interval notation. If you have an "or equals to" situation, the _____ are included as long as _____ or _____ is not in the interval. If you have more than one interval that satisfies the inequality, use the word "or" in between the inequalities in set-builder notation or use the \cup symbol to join the intervals in interval notation.

Example 1: Solve each inequality. Verify your results using a graphing calculator.

a. $x^2 + 3x - 10 > 0$

b. $6x^2 \leq 6 + 5x$

c. $2(2x^2 - 3x) > -9$

4.1: POLYNOMIAL FUNCTIONS AND MODELS

When you are done with your homework, you should be able to...

- π Identify Polynomial Functions and Their Degree
- π Graph Polynomial Functions Using Transformations
- π Identify the Real Zeros of a Polynomial Function and Their Multiplicity
- π Analyze the Graph of a Polynomial Function

WARM-UP: Use your graphing calculator to graph...

a. $f(x) = x^2$

b. $f(x) = -x^2$

c. $f(x) = x^3$

d. $f(x) = -x^3$

POLYNOMIAL FUNCTION

A polynomial function is a function of the form

Where $a_n, a_{n-1}, \dots, a_1, a_0$ are _____ numbers and n is a _____ . The domain of a polynomial function is the set of _____ numbers. The _____ of a polynomial function is the _____ of _____ that appears. The _____ polynomial function, _____ is not assigned a degree.

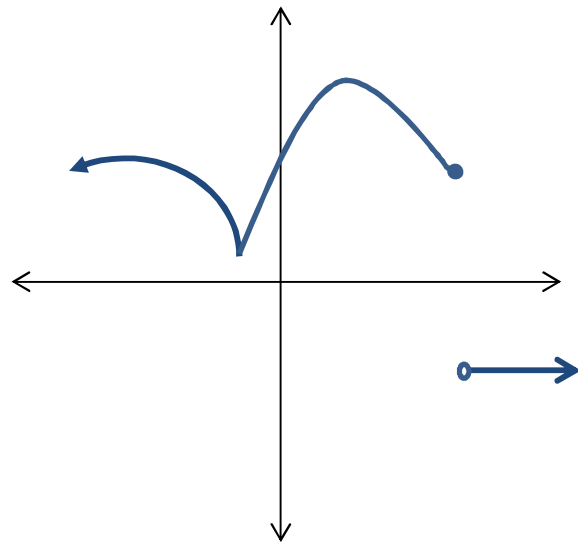
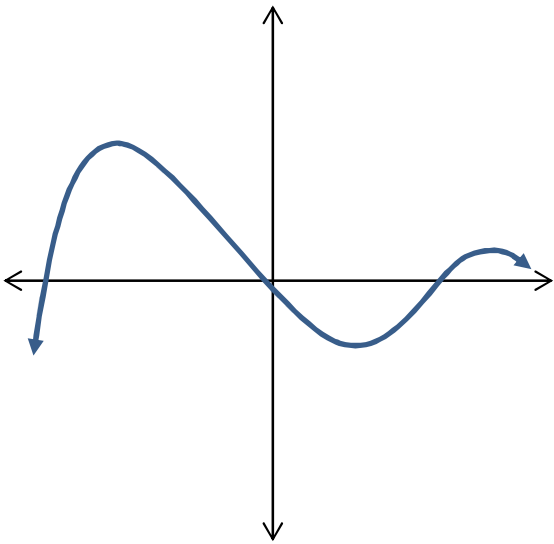
Example 1: Determine which of the following are polynomial functions. For those that are, state the degree. For those that are not, state why not.

a. $f(x) = 4x^2 - x^{2/3} + 1$

b. $g(x) = -x^{10} + \frac{3}{4}x^4 + x$

c. $f(x) = 5x^3 - x^{-2} + 10$

One objective of this section is to _____ the graph of a polynomial function. The graph of a polynomial function is both _____ and _____ . A _____ graph has no _____ corners or _____ . A _____ graph has no gaps or holes and can be drawn without lifting pencil from paper.



POWER FUNCTIONS

A power function of degree n is a _____ function of the form

Where a is a real number, _____, and _____ is an integer.

Example 2: Give three examples of power functions.

- a.
- b.
- c.

Example 3: Graph $f(x) = x^2$, $f(x) = x^4$ and $f(x) = x^{10}$ in the same window on your graphing calculator.

What do you notice about the end behavior of these graphs?

What are the x-intercept(s)?

PROPERTIES OF POWER FUNCTIONS, $f(x) = x^n$, n IS AN EVEN INTEGER

1. f is an _____ function, so its graph is symmetric with respect to the _____.
2. The domain is the set of all _____ numbers. The range is the set of all _____ numbers.
3. The graph always contains the points _____, _____, and _____.
4. As the exponent n increases in magnitude, the graph increases more rapidly when _____; but for x near the origin, the graph tends to _____ out and lie _____ to the _____.

Example 4: Graph $f(x) = x^3$, $f(x) = x^5$ and $f(x) = x^{11}$ in the same window on your graphing calculator.

What do you notice about the end behavior of these graphs?

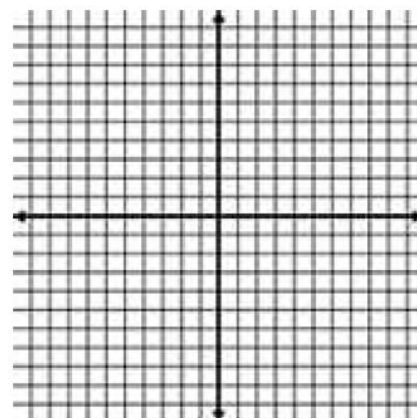
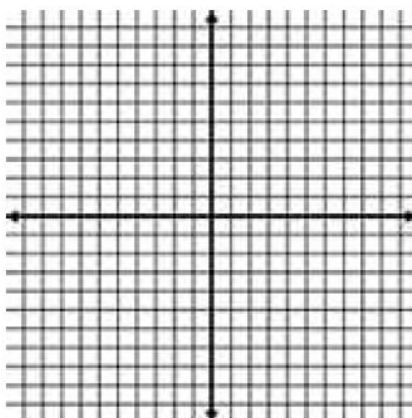
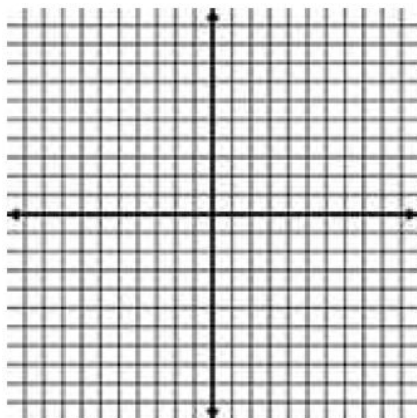
What are the x-intercept(s)?

PROPERTIES OF POWER FUNCTIONS, $f(x) = x^n$, n IS AN ODD INTEGER

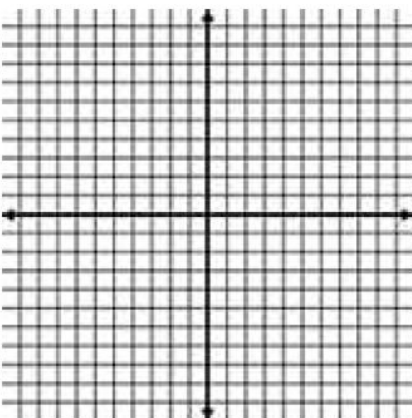
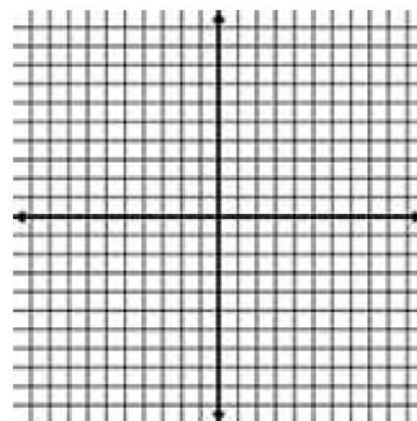
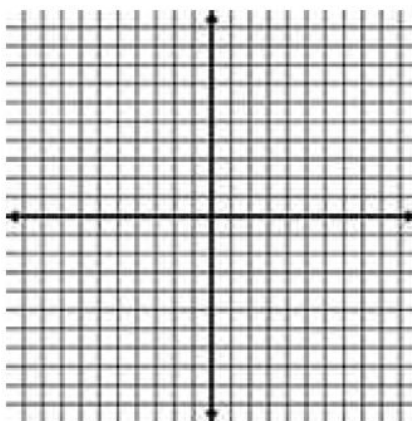
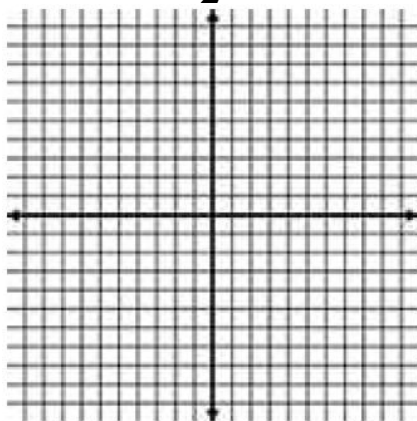
1. f is an _____ function, so its graph is symmetric with respect to the _____.
2. The domain is the set of all _____ numbers. The range is the set of all _____ numbers.
3. The graph always contains the points _____, _____, and _____.
4. As the exponent n increases in magnitude, the graph increases more rapidly when _____; but for x near the origin, the graph tends to _____ out and lie _____ to the _____.

Example 5: Graph by hand using transformations.

a. $f(x) = (2-x)^4$



b. $g(x) = \frac{1}{2}(x-1)^5 - 2$



REAL ZEROS

If f is a function and r is a real number for which $f(r) = 0$, then r is called a _____ of _____.

As a consequence of this definition, the following statements are equivalent:

1. r is a real zero of a polynomial function f .
2. r is an _____ of the graph of f .
3. $x - r$ is a _____ of f .
4. r is a solution to the equation _____.

Example 6: Form a polynomial function whose real zeros are -3, -1, 2, and 5 has degree 4.

REPEATED ZEROS

If $(x-r)^m$ is a factor of a polynomial f and $(x-r)^{m+1}$ is not a factor of f , then r is called a _____ of _____ of f .

TURNING POINTS

If f is a polynomial of degree n , then f has at most _____ turning points.

If the graph of a polynomial function f has $n - 1$ turning points, the degree of f is at least _____.

END BEHAVIOR

For _____ values of x , either positive or negative, the graph of the polynomial function

resembles the graph of the power function

Example 7: Consider the function $f(x) = (x + \sqrt{3})^2(x - 2)^4$.

- List each real zero and its multiplicity.
- Determine whether the graph crosses or touches the x -axis at each x -intercept.
- Determine the maximum number of turning points on the graph.
- Determine the end behavior.

SUMMARY: GRAPH OF A POLYNOMIAL FUNCTION

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0, \quad a \neq 0$$

Degree of the polynomial function f : _____

Graph is _____ and _____.

Maximum number of turning points: _____

At a zero of even multiplicity: The graph of f _____ the _____.

At a zero of even multiplicity: The graph of f _____ the _____.

Between zeros, the graph of f is either above or below the _____.

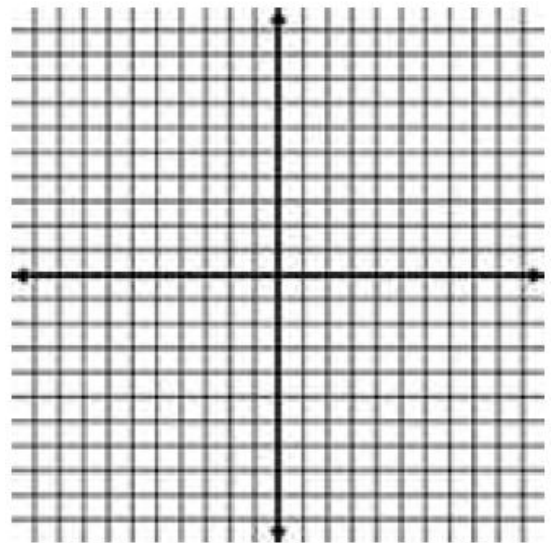
End Behavior: For large _____, the graph of f behaves like _____.

SUMMARY: ANALYZING THE GRAPH OF A POLYNOMIAL FUNCTION

1. Determine the _____ of the graph of the function.
2. Find the ___ and ___ intercepts of the graph of the function.
3. Determine the _____ of the function and their _____. Use this information to determine whether the graph _____ or _____ the x-axis.
4. Use a graphing calculator to graph the function.
5. Approximate the _____ points of the graph.
6. Use the information in steps 1-5 to draw a complete graph of the function by hand.
7. Find the _____ and _____ of the function.
8. Use the graph to determine where the function is _____ and where it is _____.

Example 8: Analyze $f(x) = x^2(x^2 + 1)(x + 4)$.

1. End Behavior.
2. Intercepts.
3. Multiplicity.
4. Use a graphing calculator to graph the function.
5. Turning points.
6. Use the information in steps 1-5 to draw a complete graph of the function by hand.
7. Domain and range.
8. Increasing/decreasing intervals.



4.2: THE REAL ZEROS OF A POLYNOMIAL FUNCTION

When you are done with your homework, you should be able to...

- π Use the Remainder and Factor Theorems
- π Use the Rational Zeros Theorem to List the Potential Rational Zeros of a Polynomial Function
- π Find the Real Zeros of a Polynomial Function
- π Solve Polynomial Equations
- π Use the Theorem for Bounds on Zeros
- π Use the Intermediate Value Theorem

WARM-UP: Divide.

$$\frac{x^2 - x + 1}{x - 1}$$

The numerator of the expression we just divided was the _____ in the division problem. The denominator was the _____. Our result had a _____ plus a _____ in x .

DIVISION ALGORITHM FOR POLYNOMIALS

If $f(x)$ and $g(x)$ denote polynomial functions and if $g(x)$ is a polynomial function whose degree is greater than zero, then there are _____ polynomial functions $q(x)$ and $r(x)$ such that

where $r(x)$ is either the zero polynomial or a polynomial function of degree less than that of $g(x)$.

REMAINDER THEOREM

Let f be a polynomial function. If $f(x)$ is divided by $x-c$, then the remainder is _____.

Example 1: Find the remainder if $f(x) = 2x^4 - 8x^3 - 10$ is divided by

a. $x-9$

b. $x+1$

FACTOR THEOREM

Let f be a polynomial function. Then $x-c$ is a factor of $f(x)$ **if and only if** _____.

1. If $f(c) = 0$, then _____ is a _____ of $f(x)$.

2. If $x-c$ is a factor of $f(x)$, then _____.

Example 2: Use the Factor Theorem to determine whether the function

$f(x) = 3x^3 - 6x^2 + 4x - 8$ has the factor

a. $x+1$

b. $x-2$

NUMBER OF REAL ZEROS

A polynomial function cannot have more real _____ than its _____.

RATIONAL ZEROS THEOREM

Let f be a polynomial function of degree 1 or higher of the form

$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$, $a_n \neq 0$, $a_0 \neq 0$ where each coefficient is an

_____. If _____, in lowest terms, is a rational zero of f , then p must be a _____ of a_0 and q must be a _____ of a_n .

Example 3: Determine the maximum number of real zeros and list the potential rational zeros of $f(x) = -2x^4 + 12x^3 + x^2 - 24x - 10$.

SUMMARY: STEPS FOR FINDING THE REAL ZEROS OF A POLYNOMIAL FUNCTION

1. Use the degree of the polynomial function to determine the maximum number of real _____.
2. If the polynomial function has _____ coefficients, use the _____ Zeros Theorem to identify those rational numbers that potentially can be _____.
3. Graph the polynomial function using your graphing calculator to find the best choice of potential rational zeros.
4. Use the _____ Theorem to determine if the potential rational zero is a _____. If it is, use synthetic division or long division to _____ the polynomial function. Each time that a zero (and thus a _____) is found, _____ step 4 on the _____ equation. In attempting to find the zeros, remember to use the factoring techniques that you already _____!!!

Example 4: Solve the equation $f(x) = 6x^4 - x^2 + 2$.

Every polynomial function (with real coefficients) can be _____
_____ into a _____ of _____ factors
and/or _____ factors.

A polynomial function (with real coefficients) of _____ degree has at least
_____.

BOUNDS ON ZEROS

Let f denote a polynomial function whose leading coefficient is 1.

$$f(x) = x^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0$$

A bound M on the real zeros of f is the smaller of the two numbers

where _____ means "choose the largest entry in $\{ \}$ ".

Example 5: Find a bound to the zeros of each polynomial function. Use the bounds to obtain a complete graph of f .

a. $f(x) = 3x^3 - 2x^2 + x + 4$

b. $f(x) = 4x^4 - 12x^3 + 27x^2 - 54x + 81$

INTERMEDIATE VALUE THEOREM

Let f denote a continuous function. If $a < b$ and if $f(a)$ and $f(b)$ are of _____ sign, then f has at least one zero between _____ and _____.

Example 6: Use the Intermediate Value Theorem to show that the function $f(x) = x^5 - 3x^4 - 2x^3 + 6x^2 + x + 2$ has a zero on the closed interval $[1.7, 1.8]$. Approximate the zero rounded to two decimal places.

Example 7: Find the real zeros of f . Use the real zeros to factor f .

a. $f(x) = x^3 + 8x^2 + 11x - 20$

b. $f(x) = 4x^4 + 15x^2 - 4$

Example 8: Find the real solutions of each equation.

a. $2x^3 - 11x^2 + 10x + 8 = 0$

b. $x^4 - 2x^3 + 10x^2 - 18x + 9 = 0$

4.3: COMPLEX ZEROS: THE FUNDAMENTAL THEOREM OF ALGEBRA

When you are done with your homework, you should be able to...

- π Use the Conjugate Pairs Theorem
- π Find a Polynomial Function with Specified Zeros
- π Find the Complex Zeros of a Polynomial Function

WARM-UP: Find all complex zeros of $f(x) = 5x^2 - x + 2$.

COMPLEX POLYNOMIAL FUNCTIONS

A variable in the _____ number system is referred to as a complex variable. A _____ function f of degree n is of the form

where $a_n, a_{n-1}, \dots, a_1, a_0$ are complex numbers, $a_n \neq 0$, n is a nonnegative integer, and x is a complex variable. As before, a_n is called the leading coefficient of f . A complex number r is called a complex zero of f if _____.

FUNDAMENTAL THEOREM OF ALGEBRA

Every _____ polynomial function $f(x)$ of degree $n \geq 1$ has at least one complex _____.

Every complex polynomial function $f(x)$ of degree $n \geq 1$ can be factored into n linear factors (not necessarily _____) of the form

where $a_n, r_1, r_2, \dots, r_n$ are complex numbers. That is, _____ complex polynomial function of degree $n \geq 1$ has _____ n complex _____, some of which may _____.

CONJUGATE PAIRS THEOREM

Let $f(x)$ be a polynomial function whose coefficients are real numbers. If _____ is a zero of f , the complex conjugate _____ is also a zero of f .

Example 1: Find a polynomial function f of degree 4 whose coefficients are real numbers and that has the zeros -2 , 1 , and $2-i$. Use your graphing calculator to verify your result.

Example 2: Find the complex zeros of each polynomial function. Write f in factored form.

a. $f(x) = x^4 - 1$

b. $f(x) = x^3 + 13x^2 + 57x + 85$

4.4: PROPERTIES OF RATIONAL FUNCTIONS

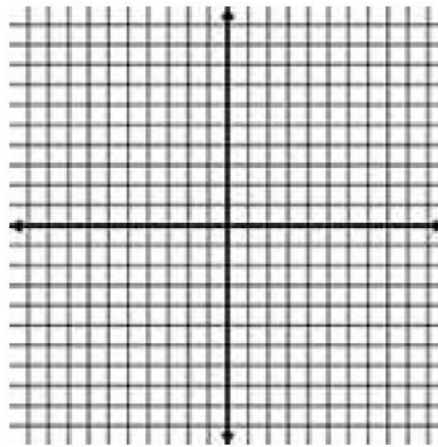
When you are done with your homework, you should be able to...

- π Find the Domain of a Rational Function
- π Find the Vertical Asymptotes of a Rational Function
- π Find the Horizontal or Oblique Asymptote of a Rational Function

WARM-UP: Graph $f(x) = \frac{1}{x}$.

What is the domain?

What is the range?



RATIONAL FUNCTIONS

A rational function is a function of the form

where p and q are polynomial functions and q is not the _____ polynomial. The domain of a rational function is the set of all _____ numbers except those for which the _____ is _____.

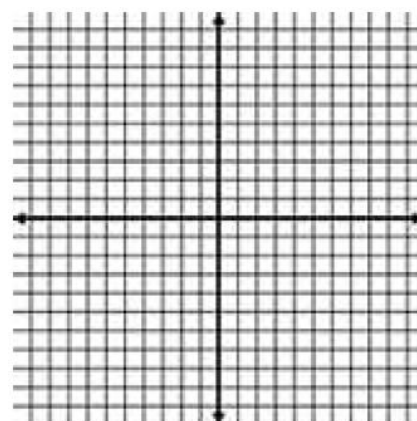
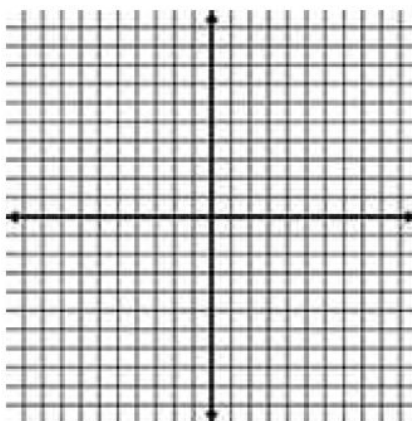
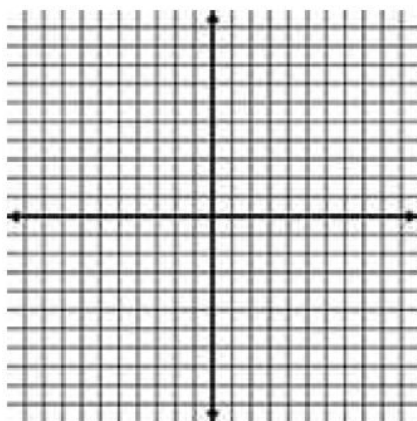
Example 1: Find the domain of the following rational functions.

a. $R(x) = \frac{5x^2}{3+x}$

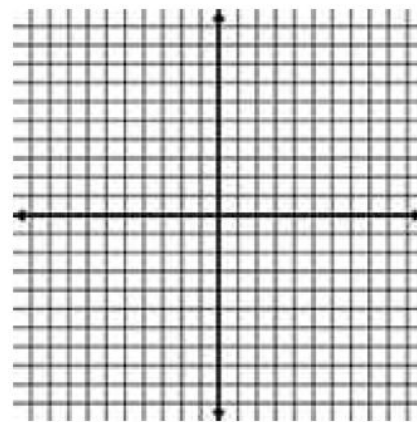
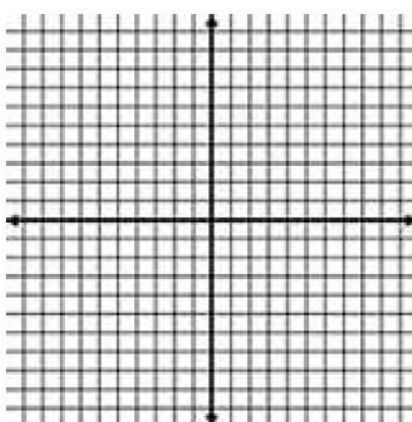
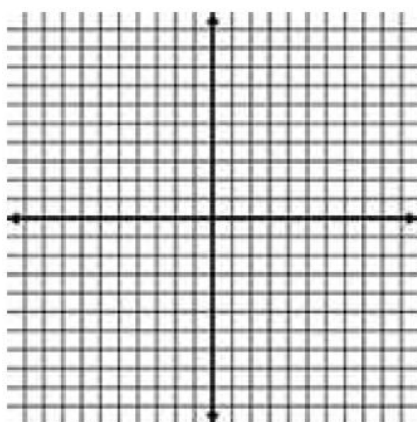
b. $F(x) = \frac{-x(1-x)}{3x^2+5x-2}$

Example 2: Graph the rational function using transformations.

a. $R(x) = \frac{1}{x-1} + 1$



b. $G(x) = \frac{-2}{x^2 - 6x + 9}$



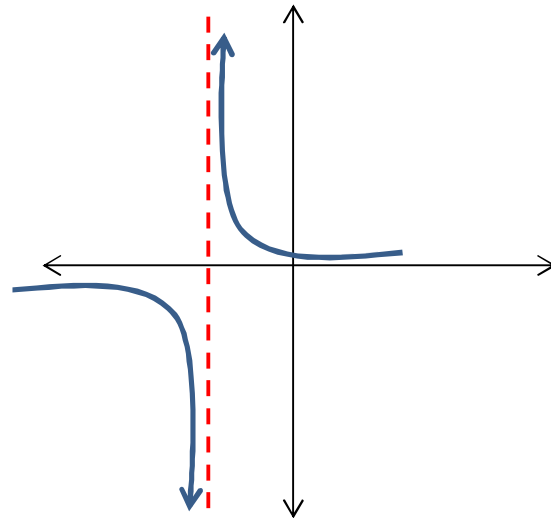
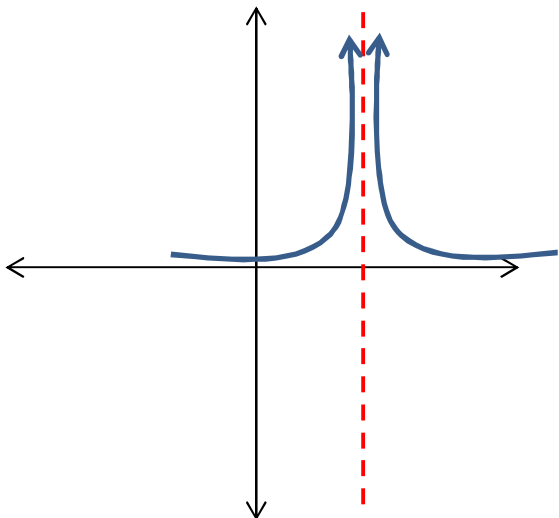
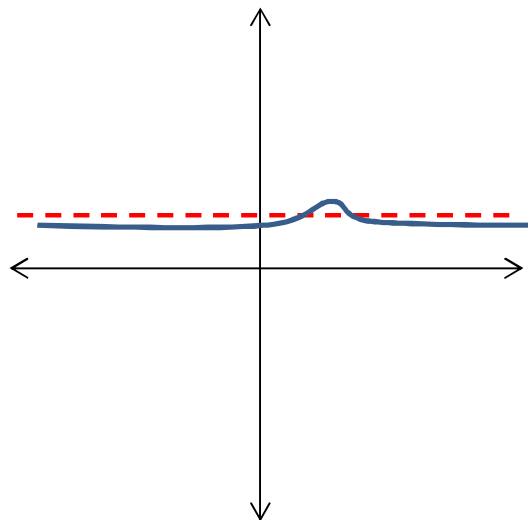
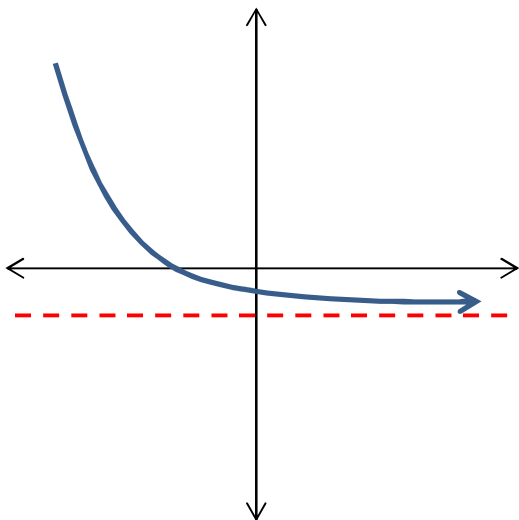
HORIZONTAL AND VERTICAL ASYMPTOTES

Let R denote a function:

If, as _____ or as _____, the values of $R(x)$ approach some _____ number _____, then the line _____ is a _____ asymptote of the graph of R .

If, as x approaches some number c , the values _____, then the line _____ is a _____ asymptote of the graph of R .

The graph of R _____ intersects a vertical asymptote!!!



LOCATING VERTICAL ASYMPTOTES

A rational function $R(x) = \frac{p(x)}{q(x)}$ in _____ terms, will have a vertical asymptote _____ if r is a real zero of the _____.

Example 3: Find the vertical asymptotes, if any, of the graph of each rational function.

a. $F(x) = \frac{x-1}{x^2+4}$

b. $Q(x) = \frac{x}{x^2+12x+32}$

c. $G(x) = \frac{x+10}{x^2-100}$

HORIZONTAL AND OBLIQUE ASYMPTOTES

To find horizontal and oblique asymptotes, we need to check out the _____ behavior of the function. If a rational function $R(x)$ is _____, that is, the degree of the numerator is _____ than the degree of the denominator, then as _____, or as _____ the value of _____ approaches _____. It follows that the line _____ is a _____ asymptote of the graph. If a rational function is _____, that is, the degree of the numerator is _____

than or _____ to the denominator, we write the rational function as the sum of a polynomial function $f(x)$ plus a proper rational function $\frac{r(x)}{q(x)}$ using long division. That is _____, where $f(x)$ is a polynomial function and $\frac{r(x)}{q(x)}$ is a proper rational function. Since $\frac{r(x)}{q(x)}$ is proper, then _____ as _____ or as _____. As a result,

So we have three possibilities:

1. If _____, a constant, then the line _____ is a _____ asymptote of the graph of R .
2. If _____, _____, then the line _____ is an _____ asymptote of the graph of R .
3. In all other cases, the graph of _____ approaches the graph of _____, and there are no horizontal or oblique asymptotes.

Example 4: Find the vertical, horizontal, and oblique asymptotes, if any, of each rational function.

a. $R(x) = \frac{7x-8}{6x+1}$

b. $F(x) = \frac{x^3+4}{2x^2-x+6}$

c. $G(x) = \frac{4x}{9x^2-121}$

4.5: THE GRAPH OF A RATIONAL FUNCTION

When you are done with your homework, you should be able to...

- π Analyze the Graph of a Rational Function
- π Solve Applied Problems Involving Rational Functions

WARM-UP: Find the zeros of $R(x) = \frac{5x^2 + 3x - 4}{x^2 + 1}$. Give both the exact result and then round to the nearest tenth.

SUMMARY: ANALYZING THE GRAPH OF A RATIONAL FUNCTION

1. _____ the numerator and denominator of R . Find the _____ of the rational function.
2. Write R in _____ terms.
3. Locate the intercepts of the graph. The _____, if any, of $R(x) = \frac{p(x)}{q(x)}$ in lowest terms satisfy the equation _____. The _____, if there is one, is _____.
4. Locate the _____ of the function. The _____, if any, of $R(x) = \frac{p(x)}{q(x)}$ in lowest terms are found by identifying the _____ of _____.
Each _____ of the _____ gives rise to a _____ asymptote.
5. Locate the _____ or _____ asymptote, if one exists. Determine _____, if any, at which the graph of R intersects this asymptote. (See Section 4.4, if you forgot 😊)
6. Use a graphing calculator to graph the function.
7. Use the information in steps 1-6 to draw a complete graph of the function by hand.

Example 1: Analyze $R(x) = \frac{2x+4}{x-1}$. If the rational function does not have a characteristic listed below, write "none".

1. Factor and identify the domain of R .

2. Write R in lowest terms.

3. Find the

a. x -intercept(s)

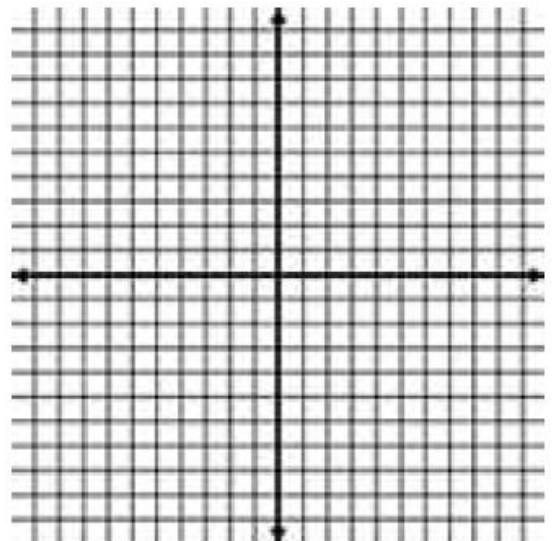
b. y -intercept

4. Locate the vertical asymptote(s).

5. Locate the horizontal or oblique asymptote.

6. Use a graphing calculator to graph the function.

7. Use the information in steps 1-6 to draw a complete graph of the function by hand.



Example 2: Analyze $R(x) = \frac{x^2 + 3x - 10}{x^2 + 8x + 15}$. If the rational function does not have a characteristic listed below, write "none".

1. Factor and identify the domain of R .

2. Write R in lowest terms.

3. Find the

a. x-intercept(s)

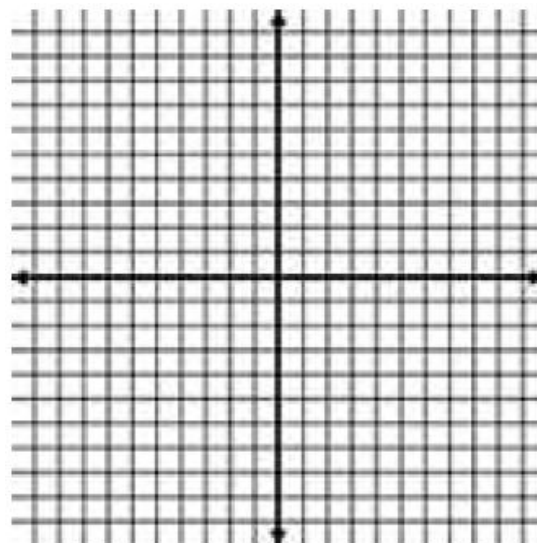
b. y-intercept

4. Locate the vertical asymptote(s).

5. Locate the horizontal or oblique asymptote.

6. Use a graphing calculator to graph the function.

7. Use the information in steps 1-6 to draw a complete graph of the function by hand.



Example 3: Analyze $R(x) = 2x + \frac{9}{x}$. If the rational function does not have a characteristic listed below, write "none".

1. Factor and identify the domain of R .

2. Write R in lowest terms.

3. Find the

a. x -intercept(s)

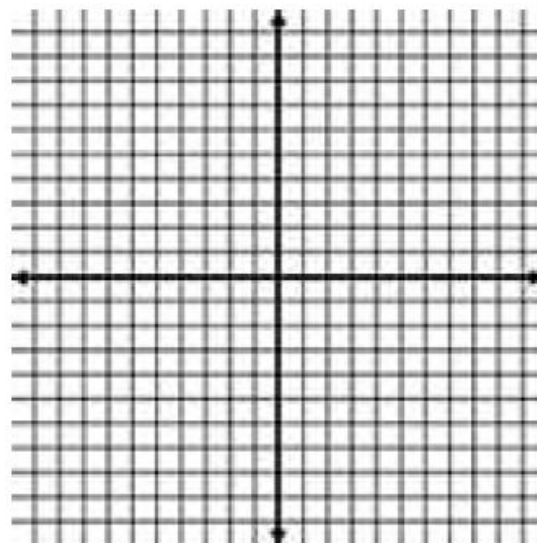
b. y -intercept

4. Locate the vertical asymptote(s).

5. Locate the horizontal or oblique asymptote.

6. Use a graphing calculator to graph the function.

7. Use the information in steps 1-6 to draw a complete graph of the function by hand.



APPLICATION

1. The concentration C of a certain drug in a patient's bloodstream t minutes after injection is given by $C(t) = \frac{50t}{t^2 + 25}$.

a. Find the horizontal asymptote of $C(t)$. What happens to the concentration of the drug as t increases?

b. Using your graphing calculator, graph $C = C(t)$.

c. Determine the time at which the concentration is highest.

4.6: POLYNOMIAL AND RATIONAL INEQUALITIES

When you are done with your homework, you should be able to...

π Solve Polynomial Inequalities Algebraically and Graphically

π Solve Rational Inequalities Algebraically and Graphically

WARM-UP: Find the domain of $R(x) = \frac{x^2 - x - 3}{4x^2 - 1}$.

STEPS FOR SOLVING POLYNOMIAL AND RATIONAL INEQUALITIES

1. Write the inequality so that a polynomial or rational expression f is on the left side and _____ is on the right side in one of the following forms:

For rational expressions, be sure that the left side is written as a _____ quotient AND find the _____ of f .

2. Determine the real numbers at which the expression f equals _____ and, if the expression is rational, the real numbers at which the expression f is _____.
3. Use the numbers found in step 2 to separate the real _____ line into _____.
4. Select a number in each _____ and evaluate f at that number.
 - a. If the value of f is _____, then _____ for all numbers _____ in the interval.
 - b. If the value of f is _____, then _____ for all numbers _____ in the interval.

If the inequality is not strict (_____ or _____), include the solutions of _____ that are in the _____ of _____ in the solution set. Be sure to _____ values of _____ where _____ is _____.

Example 1: Solve each inequality algebraically. Verify your results using a graphing calculator.

a. $(x-5)(x+2)^2 > 0$

b. $x^4 < 9x^2$

c. $\frac{5}{x-3} < \frac{3}{x+1}$

d. $\frac{x(x^2 + 1)(x - 2)}{(x - 1)(x + 1)} \geq 0$

5.1: COMPOSITE FUNCTIONS

When you are done with your homework, you should be able to...

- π Form a Composite Function
- π Find the Domain of a Composite Function

WARM-UP: Consider the function $f(x) = \sqrt{x}$.

- a. What is the domain of f ?
- b. Evaluate
 - i. $f(16)$

 - ii. $f(a)$

 - iii. $f(5x)$

What must be true about x in this part for us to evaluate f ?

COMPOSITE FUNCTIONS

Given two functions f and g , the _____ function, denoted by $f \circ g$ (read as f _____ with g) is defined by

The domain of _____ is the set of all numbers _____ in the domain of _____ such that _____ is in the domain of f .

Example 1: Let $f(x) = -x^2 + 3$ and $g(x) = 1 - x$. Find

a. $(f \circ g)(1)$

b. $(g \circ f)(1)$

c. $(f \circ f)(-2)$

d. $(g \circ g)(-1)$

Example 2: Let $f(x) = -x^2 + 3$ and $g(x) = 1 - x$. Find

a. $(f \circ g)(x)$

b. $(g \circ f)(x)$

c. $(f \circ f)(x)$

d. $(g \circ g)(x)$

e. $\frac{f(x+h) - f(x)}{h}$

Example 3: Let $f(x) = \sqrt{x-1}$ and $g(x) = x^3$. What is the domain of $f \circ g$?

Example 4: Let $f(x) = \frac{5}{2x-7}$ and $g(x) = x+2$. What is the domain of $f \circ g$?

Example 5: Let $f(x) = x^5$ and $g(x) = \sqrt[5]{x}$. Find

a. $(f \circ g)(x)$

b. $(g \circ f)(x)$

What did you notice? What do you think this means?

Example 6: Find functions f and g so that $H(x) = (1+x^2)^6$

Example 7: Find functions f and g so that $H(x) = |5x-8|$

APPLICATION

The spread of oil leaking from a tanker is in the shape of a circle. If the radius r (in feet) of the spread after t hours is $r(t) = 200\sqrt{t}$, find the area A of the oil slick as a function of the time t .

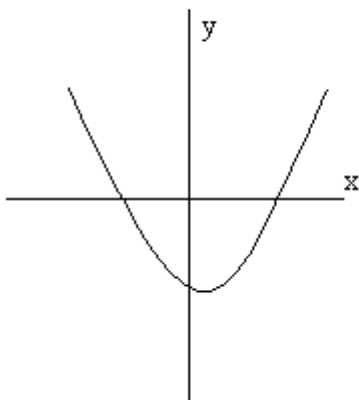
5.2: ONE-TO-ONE FUNCTIONS; INVERSE FUNCTIONS

When you are done with your homework, you should be able to...

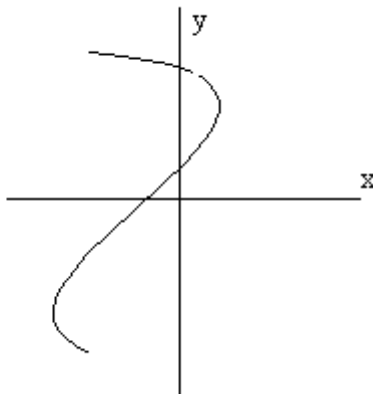
- π Determine Whether a Function is One-to-One
- π Determine the Inverse of a Function Defined by a Map or a Set of Ordered Pairs
- π Obtain the Graph of the Inverse Function from the Graph of the Function
- π Find the Inverse of a Function Defined by an Equation

WARM-UP: Use the vertical line test to determine if the graphs of the relations are functions.

a.



b.



DETERMINE WHETHER A FUNCTION IS ONE-TO-ONE

A function f is one-to-one if no _____ in the _____ is the _____ of more than one _____ in the _____. A function is not one-to-one if _____ different elements in the domain correspond to the _____ element in the range.

Example 1: Determine whether the following functions are one-to-one.

- a. For the following function, the domain represents the age of four males and the range represents the number of vehicles owned.

AGE	NUMBER OF VEHICLES
16	0
38	2
43	1
60	1

- b. $\{(-6,1),(-1,3),(0,-1),(4,8)\}$

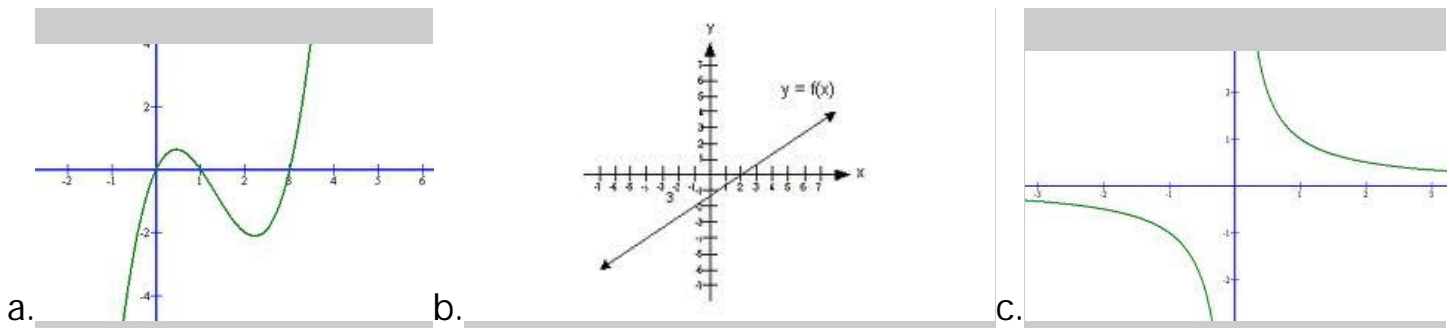
DEFINITION OF A ONE-TO-ONE FUNCTION

A function is one-to-one if and _____ inputs in the _____ correspond to _____ outputs in the _____. That is, if _____ and _____ are two different inputs of a function of f , then f is one-to-one if _____.

THEOREM: THE HORIZONTAL LINE TEST

If every _____ line intersects the graph of f in _____ one point, then f is one-to-one.

Example 2: Which of the following graphs represent one-to-one functions?



THEOREM

A function that is _____ on an interval I is a one-to-one function on I .

A function that is _____ on an interval I is a one-to-one function on I .

DEFINITION: INVERSE FUNCTION OF f

Suppose that f is a one-to-one function. Then, to each x in the domain of f , there is exactly _____ in the _____ (because f is a _____); and to each _____ in the _____ of f , there is exactly _____ in the domain (because f is _____). The _____ from the _____ of f back to the _____ of f is called the _____ function of f . The symbol _____ is used to denote the _____ of f . This symbol is read " f inverse".

Example 3: Find the inverse of each one-to-one function. State the domain and range of each inverse function.

a.

AGE	MONTHLY COST OF LIFE INSURANCE
30	\$7.09
40	\$8.40
45	\$11.29

Domain:

Range:

b. $\{(-6,1),(-1,3),(0,-1),(4,8)\}$

Domain:

Range:

TWO FACTS ABOUT A ONE-TO-ONE FUNCTION f AND ITS INVERSE f^{-1}

1. _____ of _____ = _____ of _____.

_____ of _____ = _____ of _____.

2. _____ = _____, where _____ is in the domain of _____.

_____ = _____, where _____ is in the domain of _____.

Example 4: Show that each function is the inverse of the other.

$$f(x) = 3x - 8 \text{ and } g(x) = \frac{x + 8}{3}$$

GRAPHS OF A FUNCTION AND ITS INVERSE FUNCTION

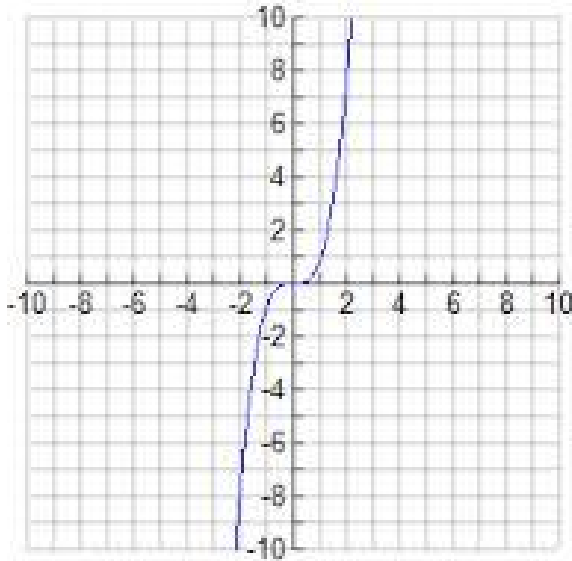
There is a _____ between the graph of a one-to-one function _____ and its inverse _____. Because inverse functions have ordered pairs with the coordinates _____, if the point _____ is on the graph of _____, the point _____ is on the graph of _____. The points _____ and _____ are _____ with respect to the line _____.

Therefore, the graph of _____ is a _____ of the graph of _____ about the line _____.

THEOREM

The graph of a one-to-one function f and the graph of its _____
 f^{-1} are _____ with respect to the line _____.

Example 5: Use the graph of f below to draw the graph of its inverse function.



POINTS ON THE GRAPH OF f	POINTS ON THE GRAPH OF f^{-1}

STEPS FOR FINDING THE INVERSE OF A FUNCTION DEFINED BY AN EQUATION

The equation of the inverse of a function f can be found as follows:

1. Replace _____ with _____ in the equation for _____.
2. Interchange _____ and _____.
3. Solve for _____. If this equation does not define _____ as a function of _____, the function _____ does not have an _____ function and this procedure ends. If this equation does define _____ as a function of _____, the function _____ has an inverse function.
4. If _____ has an inverse function, replace _____ in step 3 with _____. We can verify our result by showing that _____ and _____.

Example 6: Find an equation for $f^{-1}(x)$, the inverse function.

a. $f(x) = 4x$

b. $f(x) = \frac{2x-3}{x+1}$

APPLICATION

The function $T(g) = 1700 + 0.15(g - 17000)$ represent the 2011 federal income tax T (in dollars) due for a “married filing jointly” filer whose modified adjusted gross income is g dollars, where $17000 \leq g \leq 69000$.

- What is the domain of the function T ?
- Given that the tax due T is an increasing linear function of modified adjusted gross income g , find the range of the function T .
- Find adjusted gross income g as a function of federal income tax T . What are the domain and range of this function?

Section 5.3: EXPONENTIAL FUNCTIONS

When you are done with your homework you should be able to...

- π Evaluate Exponential Functions
- π Graph Exponential Functions
- π Define the Number e
- π Solve Exponential Equations

WARM-UP:

1. Solve. If possible, simplify radicals or rationalize denominators. Express imaginary solutions in the form $a + bi$.

$$(x^2 - 2)^2 - (x^2 - 2) = 6$$

2. Use a calculator to evaluate

a. $3^{2.2}$

b. $3^{2.24}$

c. $3^{2.236}$

d. $3^{2.2361}$

e. $3^{\sqrt{5}}$

LAWS OF EXPONENTS

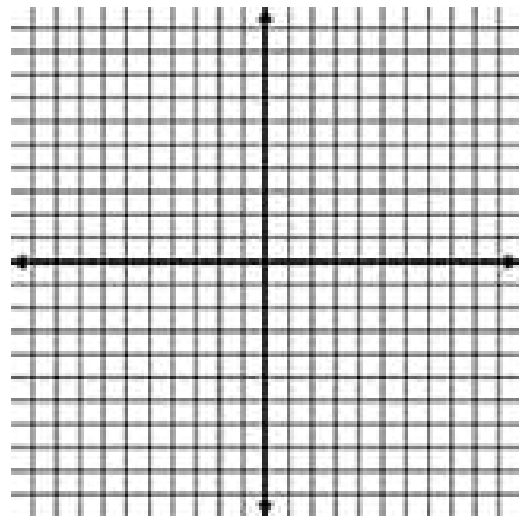
If s , t , a , and b are real numbers with _____ and _____, then

EXPONENTIAL GROWTH

Suppose a function f has the following properties:

1. The value of f doubles with every 1-unit increase in the independent variable x , and
2. The value of f at $x = 0$ is 10, so _____.

x	$f(x)$
0	10
1	
2	
3	
4	



EXPONENTIAL FUNCTIONS

An exponential function is a function of the form

where _____ is a _____ real number (_____), _____, and _____ is a real number. The domain of f all real numbers, the base a is the _____, and because _____, we call C the _____.

THEOREM

For an exponential function _____, where _____ and _____, if _____ is any real number, then

Example 1: Determine if the given function is an exponential function.

a. $f(x) = 3^x$

b. $g(x) = (-4)^{x+1}$

Example 2: Determine whether the given function is linear, exponential, or neither. For those that are linear, find a linear function that models the data. For those that are exponential, find an exponential function that models the data.

a.

x	$y = f(x)$
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4

b.

x	$y = f(x)$
-2	-1
-1	3
0	7
1	11
2	15

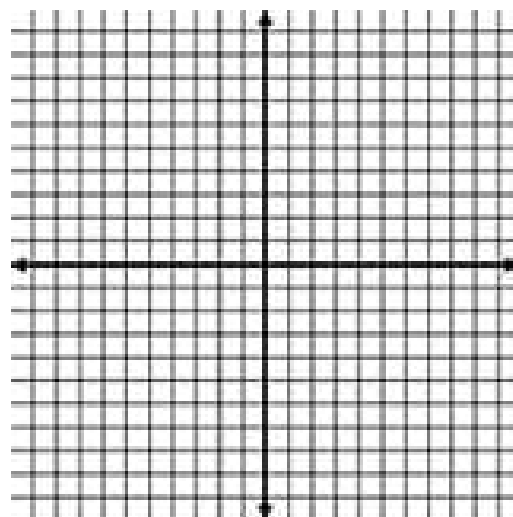
c.

x	$y = f(x)$
0	0
1	1
4	2
9	3
16	4

Example 3: Sketch the graph of each exponential function.

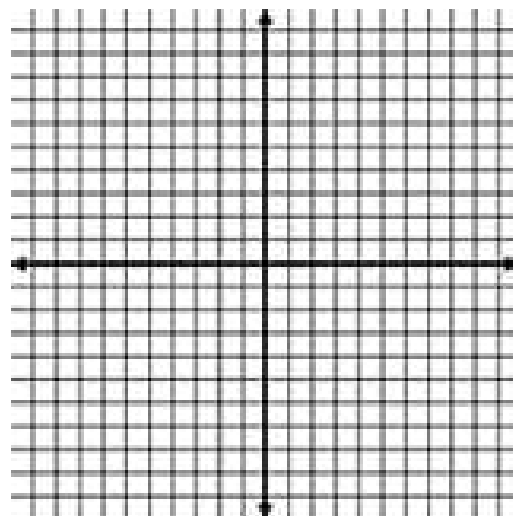
a. $f(x) = 2^x$

x	$f(x)$	$(x, f(x))$



b. $g(x) = 2^{-x}$

x	$g(x)$	$(x, g(x))$

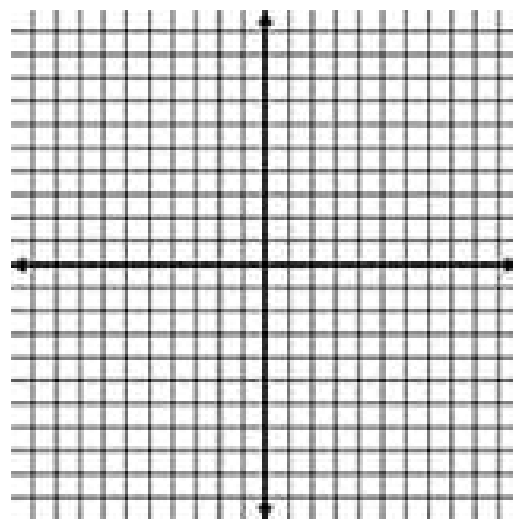


How are these two graphs related?

Example 4: Sketch the graph of each exponential function.

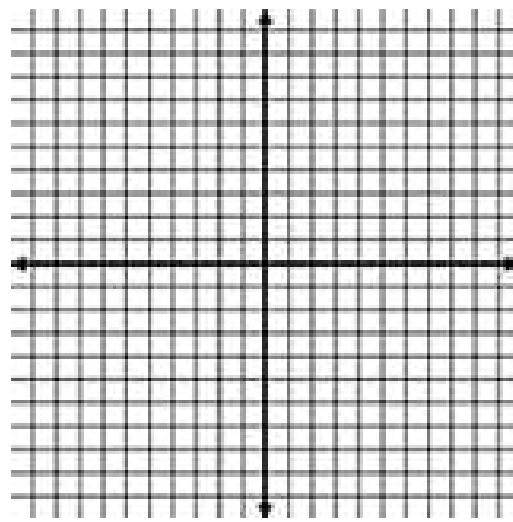
a. $f(x) = 3^x$

x	$f(x)$	$(x, f(x))$



b. $g(x) = 3^{x-1}$

x	$g(x)$	$(x, g(x))$



How are these two graphs related?

PROPERTIES OF THE EXPONENTIAL FUNCTIONS $f(x) = a^x$, $a > 1$

1. The domain is the set of all real numbers or _____ using interval notation. The range is the set of all _____ real numbers or _____ using interval notation.
2. There are no _____; The y -intercept is _____.
3. The _____ is a _____ asymptote as _____ [_____].
4. _____, where _____, is an _____ function and is _____.
5. The graph of f contains the points _____, _____, and _____.
6. The graph of f is _____ and _____, with no _____ or _____.

**Now look back at part a of each of the examples 3 and 4.

PROPERTIES OF THE EXPONENTIAL FUNCTIONS $f(x) = a^x$, $0 < a < 1$

1. The domain is the set of all real numbers or _____ using interval notation. The range is the set of all _____ real numbers or _____ using interval notation.
2. There are no _____; The y -intercept is _____.
3. The _____ is a _____ asymptote as _____ [_____].
4. _____, where _____, is an _____ function and is _____.
5. The graph of f contains the points _____, _____, and _____.
6. The graph of f is _____ and _____, with no _____ or _____.

**Now look back at example 3, part b.

$$n \quad \left(1 + \frac{1}{n}\right)^n$$

1

2

5

10

100

1000000000

The irrational number _____, approximately _____, is called the _____ base. The function _____ is called the _____ exponential function.

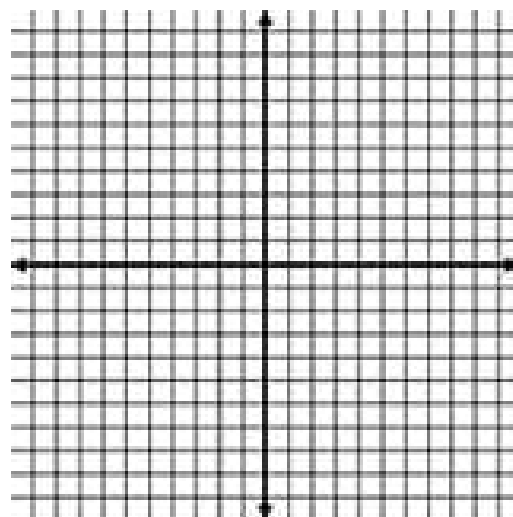
THE NUMBER e

The number e is defined as the number that the expression _____ approaches as _____. In Calculus, this is expressed using _____ notation as _____

Example 5: Sketch the graph of each exponential function.

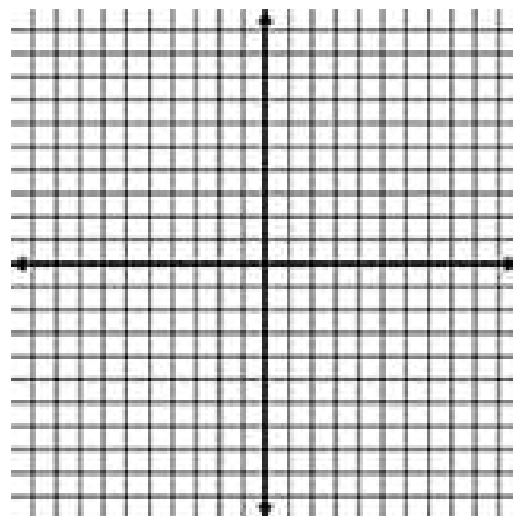
a. $f(x) = e^x$

x	$f(x)$	$(x, f(x))$



b. $g(x) = -e^x$

x	$g(x)$	$(x, g(x))$



How are these two graphs related?

EXPONENTIAL EQUATIONS

If _____, then _____.

Example 6: Solve each equation. Verify your results using a graphing calculator.

a. $8^x = 8^{-2}$

b. $9^{-x+15} = 27^x$

c. $(e^4)^x \cdot e^{x^2} = e^{12}$

APPLICATIONS

1. The normal healing of wounds can be modeled by an exponential function. If A_0 represents the original area of the wound and A equals the area of the wound, then the function $A(n) = A_0 e^{-0.35n}$ describes the area of the wound after n days following an injury when no infection is present to retard the healing. Suppose that a wound initially had an area of 100 square millimeters.
 - a. If healing is taking place, how large will the area of the wound be after 3 days?

 - b. How large will it be after 10 days?

2. Suppose that a student has 500 vocabulary words to learn. If a student learns 15 words after 5 minutes, the function $L(t) = 500(1 - e^{-0.0061t})$ approximates the number of words L that a student will learn after t minutes.
- How many words will the student learn after 30 minutes?

- How many words will the student learn after 60 minutes?

Section 5.4: LOGARITHMIC FUNCTIONS

When you are done with your homework you should be able to...

- π Change Exponential Statements to Logarithmic Statements
- π Evaluate Logarithmic Expressions
- π Determine the Domain of a Logarithmic Function
- π Graph Logarithmic Functions
- π Solve Logarithmic Equations

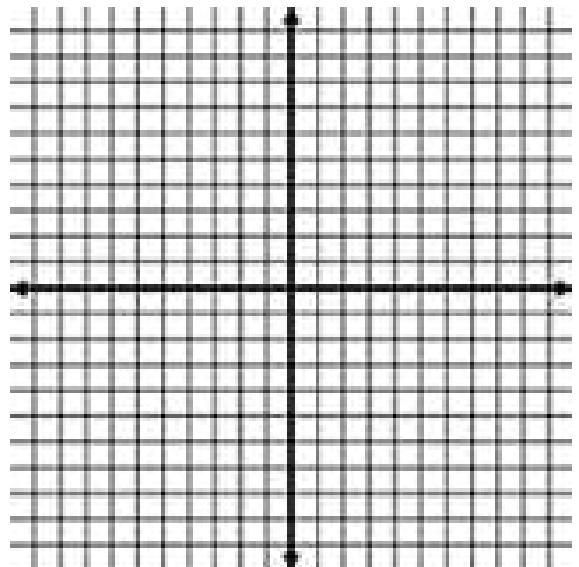
WARM-UP:

1. Solve.

$$\left(\frac{1}{64}\right)^{2x} = 16^{x^2-5}$$

2. Use the graph of $f(x) = 2^x$ to graph $f^{-1}(x)$.

POINTS ON THE GRAPH OF f	POINTS ON THE GRAPH OF f^{-1}



LOGARITHMIC FUNCTION

The logarithmic function to the base _____, where _____ and _____, is denoted by _____ (read as "_____ is the _____ to the base _____ of _____") and is defined by

The domain of the logarithmic function _____ is _____ or _____ in interval notation.

INTERESTING FACTS...

_____ of the logarithmic function = _____ of the _____ function.

_____ of the logarithmic function = _____ of the _____ function.

PROPERTIES

_____ (DEFINING EQUATION: _____)

Domain: _____ Range: _____

Example 1: Change each exponential statement to an equivalent statement involving a logarithm.

a. $16 = 4^2$

b. $e^{2.2} = M$

c. $3^x = 4.6$

Example 2: Change each logarithmic statement to an equivalent statement

involving an exponent.

a. $\log_3\left(\frac{1}{9}\right) = -2$

b. $\log_6 2 = x$

c. $\log_e x = 5$

Example 3: When working this example, remember that the expression $\log_a x$ translates as “the power to which we raise a to get x is”. Find the exact value of:

a. $\log_6\left(\frac{1}{216}\right)$

b. $\log_3 81$

c. $\log_e e$

Example 4: Find the domain of each logarithmic function.

a. $F(x) = \log_4(x+7)$

b. $h(x) = \log_e(x^2 - 16)$

c. $\log_7(-x)$

PROPERTIES OF THE LOGARITHMIC FUNCTION $f(x) = \log_a x$

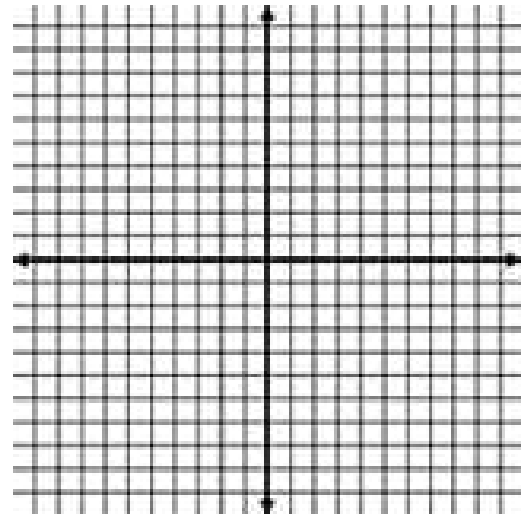
1. The domain is the set of _____ real numbers or _____ using interval notation. The range is the set of all _____ numbers or _____ using interval notation.
2. The x -intercept of the graph is _____; there is no _____.
3. The _____ (_____) is a _____ asymptote of the graph.
4. A logarithmic function is _____ if _____, and _____ if _____.
5. The graph of f contains the points _____, _____, and _____.
6. The graph of f is _____ and _____, with no _____ or _____.

**See Warm-up 2 to see the graph of $f(x) = \log_2 x$.

Example 5: Sketch the graph of the logarithmic function.

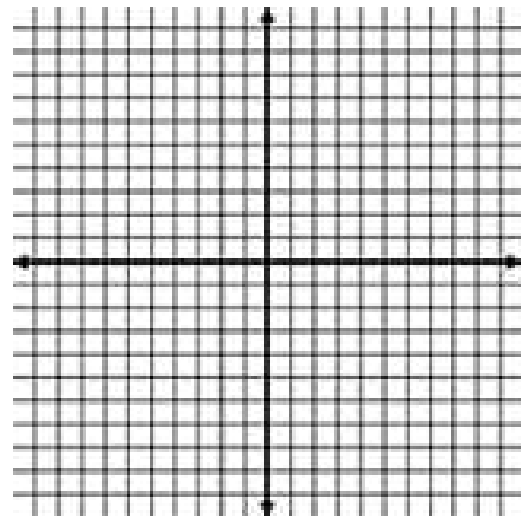
a. $f(x) = \log_3 x$

x	$f(x)$	$(x, f(x))$



b. $f(x) = \log_3(x-4)$

x	$g(x)$	$(x, g(x))$



How are these two graphs related?

If the base of a logarithmic function is _____, then we have the _____ function. We often use this function in applications. The symbol _____ denotes the natural logarithmic function. This comes from the Latin phrase *logarithmus naturalis*.

NATURAL LOGARITHMIC FUNCTION

_____ if and only if _____

If the base of a logarithmic function is the number _____, then we have the _____ function. If the base of the logarithmic function is not indicated, it is understood to be _____.

COMMON LOGARITHMIC FUNCTION

_____ if and only if _____

Example 6: Use a calculator to evaluate each expression. Round your answer to three decimal places.

a. $\frac{\ln 5}{8}$

b. $\frac{\log \frac{2}{3}}{-0.2}$

c. $\frac{\log 15 + \log 20}{\ln 15 + \ln 20}$

Example 7: Solve each equation. Verify your results using a graphing calculator.

a. $\log_5 x = 3$

b. $\log_3 (3x - 2) = 2$

c. $\ln e^{-2x} = 8$

d. $\log_6 36 = 5x + 3$

e. $\log x^2 = 4$

f. (Use your graphing calculator to solve this one☺)

$$4e^{x+1} = 5$$

2. Psychologists sometimes use the function $L(t) = A(1 - e^{-kt})$ to measure the amount L learned at time t . The number A represents the amount to be learned, and the number k measures the rate of learning. Suppose that a student has an amount A of 200 vocabulary words to learn. A psychologist determines that the student learned 20 vocabulary words after 5 minutes.
- Determine the rate of learning k .
 - Approximately how many words will the student have learned after 10 minutes?
 - After 15 minutes?
 - How long does it take for the student to learn 180 words?

Section 5.5: PROPERTIES OF LOGARITHMS

When you are done with your homework, you should be able to...

- π Work with the Properties of Logarithms
- π Write a Logarithmic Expression as a Sum or Difference
- π Evaluate a Logarithm Whose Base is Neither 10 Nor e
- π Graph a Logarithmic Function Whose Base is Neither 10 Nor e

WARM-UP:

1. Show that $\log_a 1 = 0$.

2. Show that $\log_a a = 1$.

IN SUMMARY...



PROPERTIES OF LOGARITHMS

Let ____ and ____ be positive real numbers with _____, and let ____ be any real number.

1.

2.

Example 1: Evaluate.

a. $\log_6 6$

c. $\log_9 1$

b. $\log_{12} 12^4$

d. $7^{\log_7 24}$

THE PRODUCT RULE

Let ____, ____, and ____ be positive real numbers with _____.

The logarithm of a product is the _____ of the _____.

Example 2: Expand each logarithmic expression.

a. $\log_6(6x)$

b. $\ln(x \cdot x)$

THE QUOTIENT RULE

Let _____, _____, and _____ be positive real numbers with _____.

The logarithm of a quotient is the _____ of the _____.

Example 3: Expand each logarithmic expression.

a. $\log \frac{1}{x}$

b. $\log_4 \frac{x}{2}$

THE POWER RULE

Let $_____$, $_____$, and $_____$ be positive real numbers with $_____$, and let $_____$ be any real number.

The logarithm of a power is the $_____$ of the $_____$ and the $_____$.

Example 4: Expand each logarithmic expression.

a. $\log x^2$

b. $\log_5 \sqrt{x}$

PROPERTIES FOR EXPANDING LOGARITHMIC EXPRESSIONS

For $_____$ and $_____$:

1. $_____ = \log_b M + \log_b N$

2. $_____ = \log_b M - \log_b N$

3. $_____ = p \log_b M$

Example 5: Expand each logarithmic expression.

a. $\log x^4 \sqrt[3]{y-1}$

b. $\log_2 \sqrt{\frac{x^2+5}{12y^6}}$

PROPERTIES FOR CONDENSING LOGARITHMIC EXPRESSIONS

For _____ and _____:

1. _____ = $\log_b (MN)$

2. _____ = $\log_b \frac{M}{N}$

3. _____ = $\log_b M^p$

Example 6: Write as a single logarithm.

a. $3 \ln x - \frac{1}{4} \ln(x-2)$

b. $\log_4 5 + 12 \log_4 (x+y)$

For any logarithmic bases a and b , and any positive number x , _____:

If _____, then _____.

If _____, then _____.

THE CHANGE-OF-BASE PROPERTY

If _____, _____, and _____ are positive real numbers, then

Why would we use this property?

Example 7: Use common logarithms to evaluate $\log_5 23$.

Example 8: Use natural logarithms to evaluate $\log_5 23$.

What did you find out???

APPLICATION

1. If $f(x) = \log_a x$, show that the difference quotient

$$\frac{f(x+h) - f(x)}{h} = \log_a \left(1 + \frac{h}{x} \right)^{1/h}, h \neq 0.$$

Section 5.6: LOGARITHMIC AND EXPONENTIAL EQUATIONS

When you are done with your homework, you should be able to...

- π Solve Logarithmic Equations
- π Solve Exponential Equations
- π Solve Logarithmic and Exponential Equations Using a Graphing Calculator

WARM-UP:

Solve.

$$\frac{x^2 - x}{5} = \frac{2}{5}$$

SOLVING EXPONENTIAL EQUATIONS BY EXPRESSING EACH SIDE AS A POWER OF THE SAME BASE

If _____, then _____.

1. Rewrite the equation in the form _____.

2. Set _____.

3. Solve for the variable.

Example 1: Solve.

a. $10^{x^2-1} = 100$

b. $4^{x+1} = 8^{3x}$

USING LOGARITHMS TO SOLVE EXPONENTIAL EQUATIONS

1. I solate the _____ expression.
2. Take the _____ logarithm on both sides for base _____. Take the _____ logarithm on both sides for bases other than 10.
3. Simplify using one of the following properties:
4. Solve for the variable.

Example 2: Solve.

a. $e^{2x} - 6 = 32$

b. $\frac{3^{x-1}}{2} = 5$

c. $10^x = 120$

USING EXPONENTIAL FORM TO SOLVE LOGARITHMIC EQUATIONS

1. Express the equation in the form _____.
2. Use the definition of a logarithm to rewrite the equation in exponential form:
3. Solve for the variable.
4. Check proposed solutions in the _____ equation. Include in the solution set only values for which _____.

Example 3: Solve.

a. $\log_3 x - \log_3(x - 2) = 4$

b. $\log x + \log(x + 21) = 2$

USING THE ONE-TO-ONE PROPERTY OF LOGARITHMS TO SOLVE LOGARITHMIC EQUATIONS

1. Express the equation in the form _____. This form involves a _____ logarithm whose coefficient is ____ on each side of the equation.
2. Use the one-to-one property to rewrite the equation without logarithms:
3. Solve for the variable.
4. Check proposed solutions in the _____ equation. Include in the solution set only values for which _____ and _____.

Example 4: Solve.

a. $2\log_6 x - \log_6 64 = 0$

b. $\log(5x+1) = \log(2x+3) + \log 2$

Example 5: Use a graphing calculator to solve each equation. Express your answer rounded to two decimal places.

a. $e^{2x} = x + 2$

b. $\ln 2x = -x + 2$

c. $\log_2(x-1) - \log_6(x+2) = 2$

Example 6: Solve each equation. Express irrational solutions in exact form and as a decimal rounded to three decimal places.

a. $\log_2(3x+2) - \log_4 x = 3$

b. $\frac{e^x + e^{-x}}{2} = 3$

Section 5.7: FINANCIAL MODELS

When you are done with your homework, you should be able to...

- π Determine the Future Value of a Lump Sum of Money
- π Calculate Effective Rates of Return
- π Determine the Present Value of a Lump Sum of Money
- π Determine the Rate of Interest or Time Required to Double a Lump Sum of Money

WARM-UP: If you borrow \$8,000, and, after 10 months, pay off the loan in the amount of \$8,500, what per annum rate of interest was charged?

SIMPLE INTEREST FORMULA

If a principal of P dollars is borrowed for a period of t years at a per annum interest rate r , expressed as a _____, the interest I charged is

*** Simple Interest*

FORMULAS FOR COMPOUND INTEREST

After _____ years, the balance _____, in an account with principal _____ and annual interest rate _____ (in decimal form) is given by the following formulas:

1. For _____ compounding interest periods per year:
2. For continuous compounding:

** A is referred to as the _____ value of the account and P is referred to as the _____ value.

Example 1: Find the accumulated value of an investment of \$5000 for 10 years at an interest rate of 6.5% if the money is

- a. compounded semiannually:
- b. compounded monthly:
- c. compounded continuously:

EFFECTIVE RATE OF INTEREST

The _____ of _____ of an investment earning an annual interest rate _____ is given by

Compounding _____ times per _____:

_____:

Example 2: Find the principal needed now to get each amount; that is, find the present value.

a. To get \$800 after $3\frac{1}{2}$ years at 7% compounded monthly.

b. To get \$800 after $3\frac{1}{2}$ years at 7% compounded continuously.

Example 3: Find the effective rate of interest.

a. For 5% compounded quarterly.

b. For 5% compounded continuously.

Example 4: Determine the rate that represents the better deal.

9% compounded quarterly or 8.8% compounded daily.

Section 5.8: EXPONENTIAL GROWTH AND DECAY MODELS; NEWTON'S LAW; LOGISTIC GROWTH AND DECAY MODELS

When you are done with your homework, you should be able to...

- π Find Equations of Populations That Obey the Law of Uninhibited Growth
- π Find Equations of Populations That Obey the Law of Decay
- π Use Newton's Law of Cooling
- π Use Logistic Models

WARM-UP: Graph $A(t) = 500e^{0.02t}$ and $A(t) = 500e^{-0.02t}$ on your graphing calculator. How are these graphs related?

EXPONENTIAL GROWTH AND DECAY MODELS

The mathematical model for **exponential growth** or **decay** is given by

- If _____, the function models the amount, or size, of a _____ entity. _____ is the _____ amount, or size, of the growing entity at time _____, _____ is the amount at time _____, and _____ is a constant representing the _____ rate.
- If _____, the function models the amount, or size, of a _____ entity. _____ is the _____ amount, or size, of the decaying entity at time _____, _____ is the amount at time _____, and _____ is a constant representing the _____ rate.

Example 1: A culture of bacteria obeys the law of uninhibited growth.

- a. If N is the number of bacteria in the culture and t is the time in hours, express N as a function of t .

- b. If 500 bacteria are present initially, and there are 800 after 1 hour, how many will be present in the culture after 5 hours?

Example 2: The population of a Midwestern city follows the exponential law.

- a. If N is the population of the city and t is time in years, express N as a function of t .

- b. If the population doubled in size over an 18-month period and the current population is 20,000, what will the population be 2 years from now?

Example 3: A bird species in danger of extinction has a population that is decreasing exponentially. Five years ago the population was at 1400 and today only 1000 of the birds are alive. Once the population drops below 100, the situation will be irreversible. When will this happen?

Example 4: A fossilized leaf contains 70% of its normal amount of carbon 14.

a. How old is the fossil? Use 5700 years as the half-life of carbon 14.

b. Using your graphing calculator, graph the relation between the percentage of carbon 14 remaining and time.

c. Using INTERSECT determine the time that elapses until half of the carbon 14 remains.

d. Verify the answer in part a.

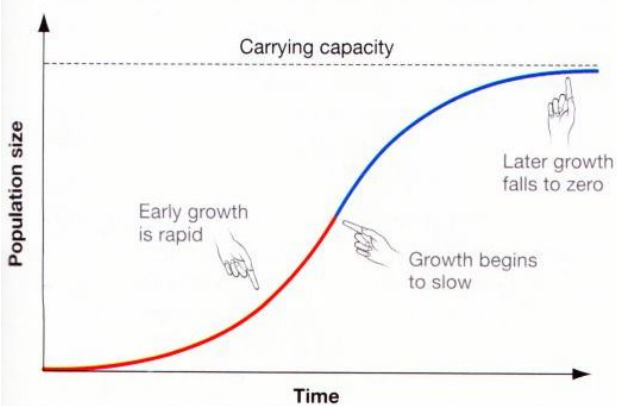
- c. Using your graphing calculator, graph the relation between temperature and time.
- d. Using INTERSECT determine the time needed to elapse before the thermometer reads 45°F .
- e. TRACE the function for large values of time. What do you notice about the temperature, y ?

LOGISTIC MODEL

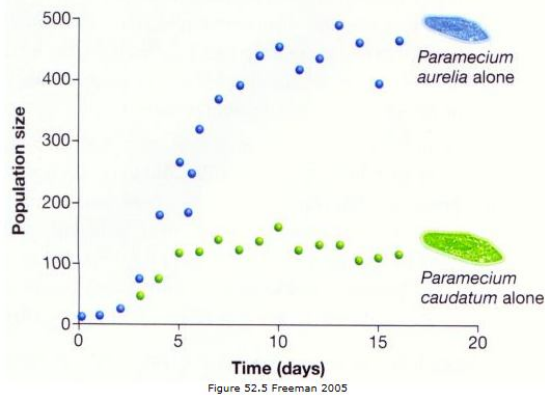
In a Logistic model, the population P after time t is given by function:

where a , b , and c are constants with _____ and _____. The model is a growth model if _____; the model is a _____ model if _____.

(a) Density dependence: growth rate is a function of population size.



(b) Data from laboratory experiments



PROPERTIES OF THE LOGISTIC MODEL

1. The domain is the set of all _____ numbers. The range is the interval _____, where c is the _____.
2. There are no _____; the _____ is _____.
3. There are _____ asymptotes:
_____ and _____.
4. _____ is an _____ function if _____ and a _____ function if _____.
5. There is an _____ point where _____ equals _____ of the carrying capacity. The inflection point is the point on the graph where the graph changes from being curved _____ to _____ for _____ functions and the point where the graph changes from being curved _____ to _____ for _____ functions.

Section 11.1: SYSTEMS OF LINEAR EQUATIONS; SUBSTITUTION AND ELIMINATION

When you are done with your homework you should be able to...

- π Solve Systems of Linear Equations by Substitution
- π Solve Systems of Linear Equations by Elimination
- π Identify Inconsistent Systems of Equations Containing Two Variables
- π Express the Solution of a System of Dependent Equations Containing Two Variables
- π Solve Systems of Three Equations Containing Three Variables
- π Identify Inconsistent Systems of Equations Containing Three Variables
- π Express the Solution of a System of Dependent Equations Containing Three Variables

WARM-UP:

Graph $5x + 3y = 21$ and $-x + 2y = 0$ using your graphing calculator.

What is the point of intersection?

What math problem have you solved?

SYSTEMS OF LINEAR EQUATIONS AND THEIR SOLUTIONS

We have seen that all _____ in the form _____ are straight _____ when graphed. _____ such equations are called a _____ of _____ or a _____ . A _____ to a system of two _____ equations in two _____ is an _____ that _____ equations in the _____ .

Example 1: Determine whether the given ordered pair is a solution of the system.

a.

$$(-2, -5)$$

$$6x - 2y = -2$$

$$3x + y = -11$$

b.

$$(10, 7)$$

$$6x - 5y = 25$$

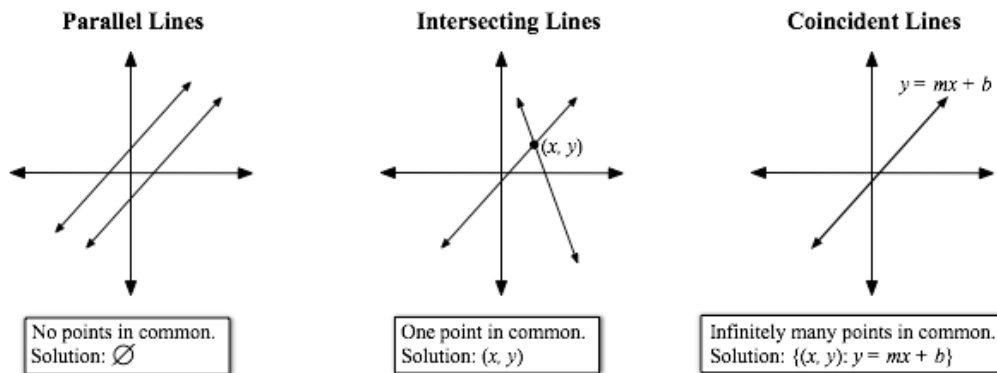
$$4x + 15y = 13$$

SOLVING LINEAR SYSTEMS BY GRAPHING

The _____ of a _____ of linear equations consists of values for the _____ that are _____ of each _____ in the _____. To _____ a system means to find _____ solutions of the _____.

TYPES OF SOLUTIONS

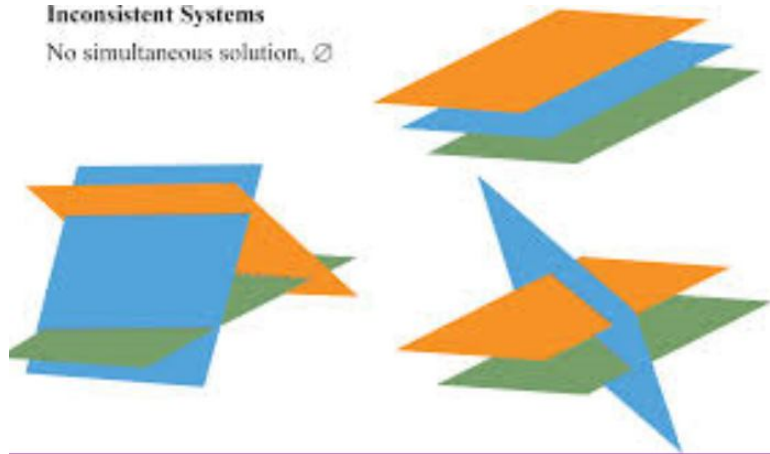
2 Equations, 2 Variables



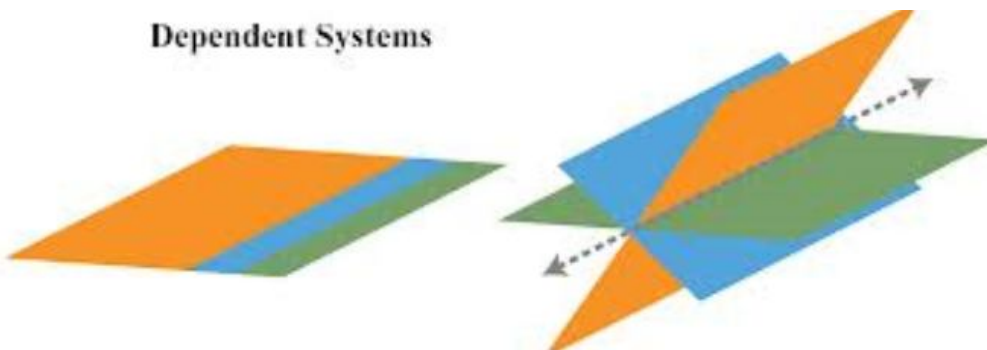
3 Equations, 3 Variables



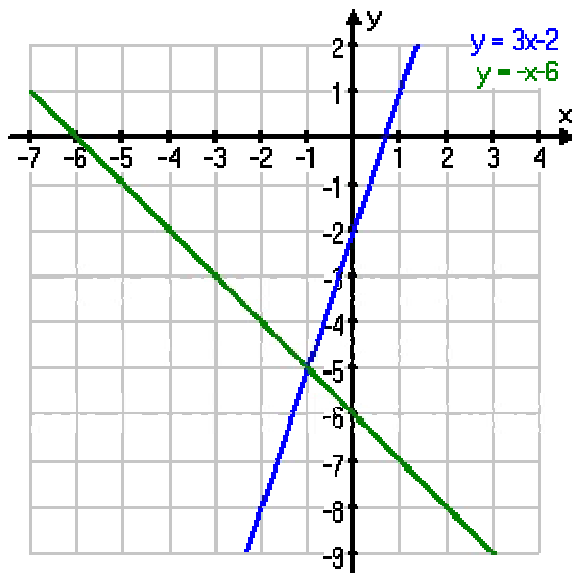
Inconsistent Systems
No simultaneous solution, \emptyset



Dependent Systems



Example 2: Use the graph below to find the solution of the system of linear equations.



RULES FOR OBTAINING AN EQUIVALENT SYSTEM OF EQUATIONS

1. _____ any two equations of the system.
2. _____ or _____ each side of an equation by the same _____ constant.
3. _____ any equation in the system by the _____ or _____ of that equation and a _____ multiple of any other equation in the system.

Example 3: Solve the following systems of linear equations by substitution. State whether the system is consistent or inconsistent. For those systems that are consistent, state whether the equations are dependent or independent.

a.

$$5x + 2y = -5$$

$$3x - y = -14$$

b.

$$y = 5x - 3$$

$$y = 2x - \frac{21}{5}$$

c.

$$-x + 3y = 4$$

$$2x - 6y = -8$$

Example 3: Solve the following systems of linear equations by the elimination method. State whether the system is consistent or inconsistent. For those systems that are consistent, state whether the equations are dependent or independent. Use set notation to express solution sets.

a.

$$x + y = 6$$

$$x - y = -2$$

b.

$$3x - y = 11$$

$$2x + 5y = 13$$

c.

$$4x - 2y = 2$$

$$2x - y = 1$$

Example 4: Solve each system of equations.

a.

$$\begin{cases} 2x + y = -4 \\ -2y + 4z = 0 \\ 3x - 2z = -11 \end{cases}$$

b.

$$\begin{cases} x - y + z = -4 \\ 2x - 3y + 4z = -15 \\ 5x + y - 2z = 12 \end{cases}$$

APPLICATIONS

1. The length of a fence required to enclose a rectangular field is 3000 meters. What are the dimensions of the field if it is known that the difference between its length and width is 50 meters?

2. A movie theater charges \$9 for adults and \$7 for students. On a day when 325 people paid an admission, the total receipts were \$2495. How many who paid were adults? How many were students?

3. Kelly has \$20000 to invest. As her financial planner, you recommend that she diversify into three investments: Treasury bills that yield 5% simple interest, Treasury bonds that yield 7% simple interest, and corporate bonds that yield 10% simple interest. Kelly wishes to earn \$1390 per year in income. Also, Kelly wants her investment in Treasury bills to be \$3000 more than her investment in corporate bonds. How much money should Kelly place in each investment?

4. The average airspeed of a single-engine aircraft is 150 mph. If the aircraft flew the same distance in 2 hours with the wind as it flew in 3 hours against the wind, what was the wind speed?

5. Find real numbers a , b , and c so that the graph of the function $y = ax^2 + bx + c$ contains the points $(-1, -2)$, $(1, -4)$, and $(2, 4)$.

11.5: PARTIAL FRACTION DECOMPOSITION

When you are done with your homework, you should be able to...

π Decompose $\frac{P}{Q}$, where Q Has Only Nonrepeated Linear Factors

π Decompose $\frac{P}{Q}$, where Q Has Repeated Linear Factors

π Decompose $\frac{P}{Q}$, where Q Has a Nonrepeated Irreducible Quadratic Factor

π Decompose $\frac{P}{Q}$, where Q Has a Repeated Irreducible Quadratic Factor

WARM-UP:

Add $\frac{3}{x(x-1)^2}$ and $\frac{5}{x-1}$.

(CASE 1) Q HAS ONLY NONREPEATED LINEAR FACTORS

Under the assumption that Q has only _____ linear factors, the polynomial Q has the form

where no two of the numbers _____ are equal. In this case, the partial fraction decomposition of _____ is of the form

where the numbers _____ are to be determined.

Example 1: Write the partial fraction decomposition of each rational expression.

a.

$$\frac{3x}{(x+2)(x-1)}$$

b.

$$\frac{x^2 - x - 8}{(x+1)(x^2 + 5x + 6)}$$

(CASE 2) Q HAS REPEATED LINEAR FACTORS

If the polynomial Q has a _____ linear factor, say _____, _____, n is an _____, then, in the partial fraction decomposition of _____, we allow for the terms

where the numbers _____ are to be determined.

Example 2: Write the partial fraction decomposition of each rational expression.

$$\frac{x+1}{x^2(x-2)}$$

(CASE 3) Q CONTAINS A NONREPEATED IRREDUCIBLE QUADRATIC FACTOR

If Q contains a _____ irreducible quadratic factor of the form _____, then, in the partial fraction decomposition of _____, allow for the term

where the numbers _____ are to be determined.

Example 3: Write the partial fraction decomposition of each rational expression.

$$\frac{1}{(x^2 + 4)(x + 1)}$$

(CASE 4) Q CONTAINS A REPEATED IRREDUCIBLE QUADRATIC FACTOR

If the polynomial Q contains a _____ irreducible quadratic factor of the form _____, _____, n is an _____, then, in the partial fraction decomposition of _____, allow for the terms

where the numbers _____ are to be determined.

Example 4: Write the partial fraction decomposition of each rational expression.

a.

$$\frac{x^3 + 1}{(x^2 + 16)^2}$$

b.

$$\frac{x^2 + 1}{x^3 + x^2 - 5x + 3}$$